

# DISTURBANCE ESTIMATION BASED TRACKING CONTROL FOR A ROBOTIC MANIPULATOR

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## Abstract

A tracking control algorithm based on a disturbance estimation scheme is studied. The tracking control problem is formulated as a disturbance rejection problem, with all the system nonlinearities and uncertainties lumped into disturbance. A disturbance estimation scheme is proposed, which is based on a nominal linear plant dynamics plus proportional error feedback corrections. The tracking control of a telescopic robot arm is used as an example to verify the proposed algorithm. The proposed algorithm is compared against a classical adaptive control and a sliding mode control algorithm.

## 1. Introduction

The tracking control of robotic manipulators has been extensively studied in recent years. If the parameters of the robot is completely known, feedback linearization technique or computed torque scheme can be used for control design. When system parameters are unknown and/or vary in wide ranges, adaptive/learning and robust/high gain control algorithms need to be applied.

Many adaptive robot control schemes assume that the structure of the manipulator dynamics is known and/or the unknown parameters influence the system dynamics in an affine manner [1-9]. There are several inherent difficulties associated with these approaches. First of all, these designs require knowledge of the structure of the manipulators, which may not be available. It has also been demonstrated [10-11] that these adaptive controllers may lack robustness against unmodeled dynamics, sensor noise, and other disturbances. More recently, adaptive control algorithms that require less model information were proposed [12-20]. This kind of approach adjust the controller gains based on the system performance and therefore is sometimes referred to as performance-based adaptive control. These algorithms require minimum knowledge of system structure and parameter values. However, the control signal is sometimes excessively large.

In this paper we introduce a disturbance-estimation based tracking control design. All the system uncertainties are lumped into the disturbance term. A feedforward control signal can then be computed to cancel the effect of the estimated disturbance signal. Since the model uncertainty and parameter variations are considered as part of the disturbance, exact model knowledge is not required.

The remainder of this paper is organized as follows: in section 2, a disturbance estimation scheme is presented. In section 3, the robotic tracking problem is formulated into a proper disturbance rejection problem. The model and control design of a telescopic robot system is presented in section. Simulation results of the proposed method as well as two competitive methods (adaptive control and sliding mode control) are presented in section 5. And finally conclusions are given in section 6.

## 2. Disturbance Estimation Schemes

The nonlinear system studied in this paper is assumed to be described by

$$\dot{x}(t) = Ax(t) + \Gamma(u, x) + Bd \quad (1)$$

where  $x \in R^n$  denotes the state vector,  $d \in R^m$  is the disturbance input,  $\Gamma(u, x) \in R^n$  is a known nonlinear function, and  $A \in R^{n \times n}$  is a known constant matrix,  $B \in R^{n \times m}$  is a full column rank matrix and is assumed to be known. An "observer" for this system can be described by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \Gamma(u, x) + B\hat{d}(t) + K(x - \hat{x}) \quad (2)$$

where  $\hat{\cdot}$  denotes estimated signals,  $K \in R^{n \times n}$  is the observer gain. It should be noted that this "observer" is full-state feedback, and the objective is really to estimate disturbance rather than estimating states. The error dynamics can be obtained by subtracting Eq.(1) from Eq.(2)

$$\dot{e} = A_k e + B e_d \quad (3)$$

where  $e = \hat{x} - x$ ,  $e_d = \hat{d}(t) - d(t)$ , and  $A_k = A - K$ . Since we assume full state feedback,  $K$  can be chosen so that matrix  $A_k$  becomes Hurwitz. The disturbance estimation laws are chosen to be

$$\dot{e}_{d_o} = -B^T P e \quad (4)$$

$$\hat{d}(t) = \hat{d}_o(t) - K_o e \quad (5)$$

$$K_o A_k + K_o B K_o + B^T P = 0 \quad (6)$$

where  $\hat{d}_o(t)$  is the "uncorrected" estimated disturbance,  $\hat{d}(t)$  is the "corrected" estimated disturbance, and  $K_o$  is the linear correction gain. If  $A_k$  is Hurwitz, then there exist positive definite matrices  $P$  and  $Q$  so that

$$A_k^T P + P A_k = -Q \quad (7)$$

To avoid confusion, the derivation of the estimation scheme is divided into two steps: pre-correction (for constant disturbance) and post-correction (for time varying disturbance). The observer with uncorrected disturbance estimation is

$$\dot{\hat{x}} = A\hat{x} + \Gamma(u, x) + B\hat{d}_o(t) + K(x - \hat{x}) \quad (8)$$

where  $\hat{d}_o(t)$  is the uncorrected estimated disturbance. Subtract Eq.(1) from Eq.(8), we have

$$\dot{e} = A_k e + B e_{d_o} \quad (9)$$

where  $e_{d_o} = \hat{d}_o(t) - d(t)$ .

**Fact 1:** Suppose  $A_k$  is Hurwitz, then the update law

$$\dot{e}_{d_o} = -B^T P e$$

guarantees  $\dot{V} < 0$  for the Lyapunov function  $V = e^T P e + e_{d_o}^T e_{d_o}$  when the disturbance is time invariant.

**Proof:** Cf. [21].

**Fact 2:** Suppose Fact 1 is satisfied, then the following conclusions can be made

i:  $e \in \mathcal{L}^\infty \cap \mathcal{L}^2$ , and  $e_{d_o} \in \mathcal{L}^\infty$

ii:  $\lim_{t \rightarrow \infty} e(t) \rightarrow 0$  and  $\lim_{t \rightarrow \infty} B e_{d_o}(t) \rightarrow 0$ .

If  $B$  is a full column matrix, then we also have

iii:  $\lim_{t \rightarrow \infty} e_{d_o}(t) \rightarrow 0$ .

**Proof:** Cf. [21].

**Fact 3:** If Facts 1-2 are satisfied,  $\dot{e}$ ,  $e$ , and  $e_{d_o}$  converge exponentially to zero.

**Proof:** Cf. [21].

According to Fact 3 the update law shown in Eq.(4) will guarantee the convergence of the estimation error for both state and input signals. However, since  $d(t)$  is unknown, the update law cannot be implemented unless  $d(t)$  is constant or slowly varying. To relax this restriction we need to introduce an estimation correction procedure in the following.

**Lemma 1:** Suppose Facts 1-2 are satisfied, and we assume that the disturbance estimation error is proportional to state estimation error, i.e.,  $e_{d_o} = K_o e$ . This proportional matrix  $K_o$  can be solved from the equation

$$K_o A_k + K_o B K_o + B^T P = 0$$

and the "corrected" disturbance estimation is then

$$\hat{d}(t) = \hat{d}_o(t) - K_o e$$

**Proof:** Cf. [21].

The procedures of estimating the disturbances are summarized in the block diagram shown in Figure 1.

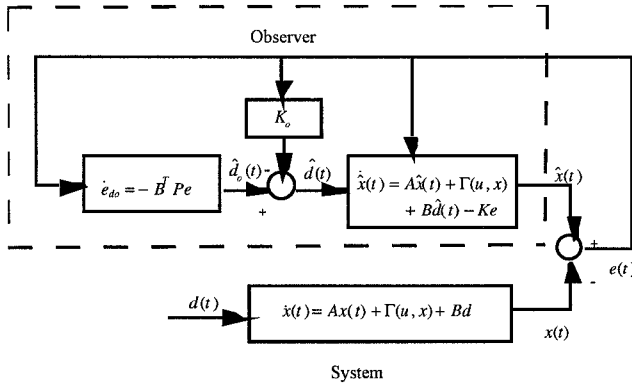


Fig. 1 Schematic Representation of Disturbance Estimation

### 3. Tracking Control of Robot Manipulators

#### 3.1 Dynamic Equations of Robot Manipulators

Nominal dynamic equations of a robot manipulator, in general, can be described by

$$M_n(q)\ddot{q} + C_n(q, \dot{q})\dot{q} + g_n(q) = F \quad (10)$$

where  $q \in \mathcal{R}^n$  denotes the generalized coordinates, the subscript  $(\cdot)_n$  denotes nominal functions,  $M_n(q) \in \mathcal{R}^{n \times n}$  is the manipulator inertia matrix,  $C_n(q, \dot{q}) \in \mathcal{R}^{n \times n}$  includes the Coriolis and centripetal acceleration effects,  $g_n(q)$  is the gravity term, and  $F \in \mathcal{R}^n$  is the control input associated with the generalized coordinate  $q$ . The dynamic equation of the true plant is assumed to be

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + d(q, \dot{q}, t) = F \quad (11)$$

where  $M(q) = M_n(q) + \Delta M(q)$ ,  $C(q, \dot{q}) = C_n(q, \dot{q}) + \Delta C(q, \dot{q})$ ,

$g(q) = g_n(q) + \Delta g(q)$  are the real system matrices.  $d(q, \dot{q}, t) \in \mathcal{R}^n$  represents the disturbance vector.

#### 3.2 Disturbance Estimation Based Tracking Control

If the parameters of the system are exactly known, we can apply the computed torque method such that

$$F = M_n(q)(\ddot{q}_d - k_1 \dot{e}_d - k_2 e_d) + C_n(q, \dot{q})\dot{q} + g_n(q) \quad (12)$$

where  $q_d$  is the desired trajectory,  $e_d = q - q_d$ ,  $k_1$  and  $k_2$  are feedback gain matrices

Substitute Eq.(12) into Eq.(10), the error dynamics becomes

$$\ddot{e}_d + k_1 \dot{e}_d + k_2 e_d = 0 \quad (13)$$

Appropriate gains  $k_1$  and  $k_2$  can be chosen so that Eq.(13) is asymptotically stable. When uncertainties exist, however, the error dynamics become

$$\ddot{e}_d + k_1 \dot{e}_d + k_2 e_d + \delta(\ddot{q}, \dot{q}, q) = 0 \quad (14)$$

where

$$\delta(\ddot{q}, \dot{q}, q) = \Delta M(q)\ddot{q} + \Delta C(q, \dot{q})\dot{q} + \Delta g(q) + d(q, \dot{q}, t) \quad (15)$$

We can see that if  $\delta(\ddot{q}, \dot{q}, q) \neq 0$ , the controller (12) can only drive the system output to a neighborhood of the desired trajectory. In the following, we will introduce a different tracking control algorithm which is referred to as the "disturbance estimation based tracking control" in this paper.

#### Lemma 2:

If we choose the control input to be

$$F = \ddot{q}_d - k_1 \dot{e}_d - k_2 e_d - \hat{w}_d(q, \dot{q}, \ddot{q}, t)$$

and the disturbance estimation  $\hat{w}_d(q, \dot{q}, \ddot{q}, t)$  is obtained from

$$\begin{aligned} \dot{\hat{w}}_{d_o} &= -B^T P E_d \\ \hat{w}_d(q, \dot{q}, \ddot{q}, t) &= \hat{w}_{d_o}(q, \dot{q}, \ddot{q}, t) - K_o e_d \end{aligned}$$

where  $E_d = \begin{bmatrix} e_d \\ \dot{e}_d \end{bmatrix}$ ,  $K_o A_d + K_o B K_o + B^T P = 0$ . Then we can have

$$\lim_{t \rightarrow \infty} e_d(t) \rightarrow 0, \lim_{t \rightarrow \infty} \dot{e}_d(t) \rightarrow 0, \text{ and } \lim_{t \rightarrow \infty} e_w(t) \rightarrow 0.$$

#### Proof:

Rewrite Eq.(11) as

$$\ddot{q} = F - (M(q) - I_{n \times n})\ddot{q} - C(q, \dot{q})\dot{q} - g_n(q) - d(q, \dot{q}, t) \quad (16)$$

where  $I_{n \times n} \in \mathcal{R}^{n \times n}$  is an identity matrix. Combine the system dynamic subject to uncertainties with the disturbance term  $d(q, \dot{q}, t)$ , we have

$$\ddot{q} = F + w_d(q, \dot{q}, \ddot{q}, t) \quad (17)$$

where

$$w_d(q, \dot{q}, \ddot{q}, t) = -(M(q) - I_{n \times n})\ddot{q} - C(q, \dot{q})\dot{q} - g_n(q) - d(q, \dot{q}, t) \quad (18)$$

The control law is then

$$F = \ddot{q}_d - k_1 \dot{e}_d - k_2 e_d - \hat{w}_d(q, \dot{q}, \ddot{q}, t) \quad (19)$$

where  $\hat{w}_d(q, \dot{q}, \ddot{q}, t)$  is the estimation of the total disturbance signal  $w_d(q, \dot{q}, \ddot{q}, t)$ . Substitute Eq.(19) into Eq.(17), the error dynamics becomes

$$\ddot{e}_d + k_1 \dot{e}_d + k_2 e_d = e_w \quad (20)$$

Eq.(20) can be presented in the state space form as

$$\dot{E}_d = A_d E_d + B e_w \quad (21)$$

where  $E_d = \begin{bmatrix} e_d \\ \dot{e}_d \end{bmatrix}$ ,  $A_d = \begin{bmatrix} 0 & I_{n \times n} \\ -k_2 & -k_1 \end{bmatrix}$ , and  $B = \begin{bmatrix} 0 \\ I_{n \times n} \end{bmatrix}$ . The feedback gains  $k_1$  and  $k_2$  can then be selected so that  $A_d$  is Hurwitz. Comparing Eq.(21) with Eq.(1), and apply Fact 1, we can see that if we choose the Lyapunov function candidate

$$V = E_d^T P E_d + e_w^T e_w \quad (22)$$

where  $P$  satisfies  $A_d^T P + P A_d = -Q$ . Then

$$\dot{e}_w = -B^T P E_d \quad (23)$$

guarantees

$$\dot{V} = -E_d^T Q E_d < 0 \quad (24)$$

Since  $B$  has full column rank, from Fact 2 we have  $\lim_{t \rightarrow \infty} E_d(t) \rightarrow 0$

and  $\lim_{t \rightarrow \infty} e_w(t) \rightarrow 0$ . Apply the modification presented in Lemma 1, the disturbance update law becomes

$$\dot{e}_{w_0} = -B^T P E_d \quad (25)$$

$$\hat{w}_d(q, \dot{q}, \ddot{q}, t) = \hat{w}_{d_0}(q, \dot{q}, \ddot{q}, t) - K_o E_d \quad (26)$$

$$K_o A_d + K_o B K_o + B^T P = 0 \quad (27)$$

where  $\hat{w}_d(q, \dot{q}, \ddot{q}, t)$  is the final (corrected) disturbance estimation used in the control law.

#### 4. Case Study ( Tracking Control of a Telescopic Robot Arm )

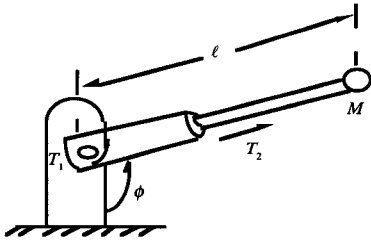


Fig. 2 Telescopic Robot Arm

##### 4.1 System Description

The schematic diagram of the telescopic robot arm is shown in Fig. 2. Two actuators were used to control the angle  $\phi$  and length  $\ell$ . It is assumed that the arm mass is small compare to the payload  $M$ . The dynamic equations were found to be

$$\frac{M\ell^2}{\alpha_m} \left( \ddot{\phi} + \alpha_1 \dot{\phi} + \alpha_2 \phi + \frac{g}{\ell} \sin(\phi) + d_1 \right) = u_1 \quad (28)$$

$$\frac{M}{k_m} \left( \ddot{\ell} + \alpha_3 \dot{\ell} + \alpha_4 \ell - g \cos(\phi) + d_2 \right) = u_2 \quad (29)$$

where  $\alpha_1 = \frac{\alpha_f}{M\ell^2} + 2\frac{\dot{\ell}}{\ell}$ ,  $\alpha_2 = \frac{\alpha_s}{M\ell^2}$ ,  $\alpha_3 = \frac{k_f}{M}$ ,  $\alpha_4 = \frac{k_s}{M} - \dot{\phi}^2$ ,  $d_1$  and  $d_2$  are unknown external disturbances,  $\alpha_s$  and  $k_s$  are the stiffness coefficients,  $\alpha_f$  and  $k_f$  are the viscous friction coefficients,  $u_1$  and  $u_2$  are the electrical currents applied to the actuators. The torque in the joint and the force in the arm are  $T_1 = \alpha_m u_1$  and  $T_2 = k_m u_2$ , respectively, where  $\alpha_m$  and  $k_m$  are constants. In the following subsections, three full-state feedback control algorithms are presented.

##### 4.2 Adaptive Control

It is intuitive to compensate the uncertainty caused by the bias between nominal parameter value and true parameter value by estimating the parameters. A classical adaptive observer [22] is applied to estimate the system parameters  $\alpha_m$ ,  $\alpha_f$ ,  $\alpha_s$ ,  $k_m$ ,  $k_f$ , and  $k_s$ . Assume that the nonlinear plant is described by

$$\dot{x} = Ax + B\theta^T f(y, u) + g(y, u) \quad (30)$$

where  $u$  is the system input,  $g(y, u)$  is a known function,  $\theta$  is the unknown parameter, and in this case study we assume full state feedback and thus  $y = x$ . The adaptive observer is chosen as

$$\dot{\hat{x}} = A\hat{x} + B\hat{\theta}^T f(y, u) + g(y, u) + K(y - \hat{y}) \quad (31)$$

where  $K$  is the observer gain, and  $\hat{\cdot}$  denotes estimation. The adaptive law to estimate the parameter  $\theta$  can be derived based on the Lyapunov function, which is given by

$$\dot{\hat{\theta}} = -f'(y, u) B^T P e \quad (32)$$

where  $e = \hat{x} - x$ ,  $P$  is a positive definite matrix that satisfies the relationship  $A_k^T P + P A_k = -Q$ ,  $Q > 0$ ,  $A_k = A - KC$  is assumed to be Hurwitz. To make sure the estimated parameters converge to true values,  $f(y, u)$  needs to be a persistent excitation function. From Eq.(32) the update laws are

$$\begin{bmatrix} \dot{\hat{\alpha}}_s & \dot{\hat{\alpha}}_f & \dot{\hat{\alpha}}_m \end{bmatrix} = - \begin{bmatrix} -\phi & -\dot{\phi} & u_1 \end{bmatrix} [0 \ 1] P_\phi \begin{bmatrix} \hat{\phi} - \phi \\ \dot{\hat{\phi}} - \dot{\phi} \end{bmatrix} \quad (33)$$

$$\begin{bmatrix} \dot{\hat{k}}_s & \dot{\hat{k}}_f & \dot{\hat{k}}_m \end{bmatrix} = - \begin{bmatrix} -\ell & -\dot{\ell} & u_2 \end{bmatrix} [0 \ 1] P_\ell \begin{bmatrix} \hat{\ell} - \ell \\ \dot{\hat{\ell}} - \dot{\ell} \end{bmatrix} \quad (34)$$

The adaptive controller can be described as

$$u_1 = \frac{M\ell^2}{\hat{\alpha}_m} \left( \ddot{\phi}_d + \hat{\alpha}_1 \dot{\phi} + \hat{\alpha}_2 \phi + \frac{g \sin(\phi)}{\ell} + k_1 \dot{e}_\phi + k_2 e_\phi \right) \quad (35)$$

$$u_2 = \frac{M}{\hat{k}_m} \left( \ddot{\ell}_d + \hat{\alpha}_3 \dot{\ell} + \hat{\alpha}_4 \ell - g \cos(\phi) + k_3 \dot{e}_\ell + k_4 e_\ell \right) \quad (36)$$

$$\text{where } \hat{\alpha}_1 = \frac{\hat{\alpha}_f}{M\ell^2} + 2\frac{\dot{\ell}}{\ell}, \hat{\alpha}_2 = \frac{\hat{\alpha}_s}{M\ell^2}, \hat{\alpha}_3 = \frac{\hat{k}_f}{M} \text{ and } \hat{\alpha}_4 = \frac{\hat{k}_s}{M} - \dot{\phi}^2$$

##### 4.3 Sliding Mode Control

Sliding mode control is widely used to design robust controllers under unmodeled dynamics and external disturbances. The basic idea of sliding mode control is to construct the control input with switching (saturation) functions with gains designed according to the bounds of uncertainties. The sliding surfaces are chosen as

$$S_1 = \dot{\phi} - \dot{\phi}_d + \lambda_1 (\phi - \phi_d) \quad (37)$$

$$S_2 = \dot{\ell} - \dot{\ell}_d + \lambda_2 (\ell - \ell_d) \quad (38)$$

where  $\lambda_1$  and  $\lambda_2 > 0$  determine the sliding dynamics. We choose to use

$$\dot{S}_1 = -\kappa_1 \text{sat}\left(\frac{S_1}{\lambda_1 \phi_b}\right) \quad (39)$$

$$\dot{S}_2 = -\kappa_2 \text{sat}\left(\frac{S_2}{\lambda_2 \ell_b}\right) \quad (40)$$

to construct the control laws, where  $\kappa_1$  and  $\kappa_2 > 0$ ,  $\text{sat}(\cdot)$  is the saturation function,  $\phi_b$  and  $\ell_b$  are the chosen boundary layer.

Sliding mode controller can compensate the system uncertainty and unknown external disturbance and drive the manipulator to follow the desired trajectory. However, sliding mode controller needs the information of system dynamics and bounds of the parameters to determine the switching function. Chatters caused by the switching function can be eliminated by introducing a boundary layer, but the accuracy of tracking will be sacrificed.

##### 4.4 Disturbance Estimation Based Tracking Control

Eqs.(28) and (29) can be rewritten as

$$\ddot{\phi} = u_1 + w_\phi \quad (41)$$

$$\ddot{\ell} = u_2 + w_\ell \quad (42)$$

$$\text{where } w_\phi = \ddot{\phi} - \frac{M\ell^2}{\alpha_m} \left( \ddot{\phi} + \alpha_1 \dot{\phi} + \alpha_2 \phi + \frac{g}{\ell} \sin(\phi) + d_1 \right)$$

$$w_t = \ddot{\ell} - \frac{M}{k_m} (\ddot{\ell} + \alpha_3 \dot{\ell} + \alpha_4 \ell - g \cos(\phi) + d_2)$$

The controller can be chosen as

$$u_1 = \ddot{\phi}_d + k_1 \dot{e}_\phi + k_2 e_\phi - \hat{w}_\phi \quad (43)$$

$$u_2 = \ddot{\ell}_d + k_3 \dot{e}_\ell + k_4 e_\ell - \hat{w}_\ell \quad (44)$$

Substitute Eqs.(43) and (44) into (41) and (42), the error dynamics become

$$\ddot{e}_\phi - k_1 \dot{e}_\phi - k_2 e_\phi = e_{w\phi} \quad (45)$$

$$\ddot{e}_\ell - k_3 \dot{e}_\ell - k_4 e_\ell = e_{w\ell} \quad (46)$$

where  $e_{w\phi} = w_\phi - \hat{w}_\phi$ , and  $e_{w\ell} = w_\ell - \hat{w}_\ell$ .

The estimation of disturbance signals  $w_\phi$  and  $w_\ell$  can be obtained by following the procedures listed in section 3. The detail equations are omitted here.

### 5. Numerical Simulation Results

In the following we will compare the simulation results of the three control algorithms presented in the previous section. The nominal parameter values are assumed to be  $\bar{\alpha}_m = \bar{k}_m = 1.0$ ,  $\bar{\alpha}_s = \bar{k}_s = 0.65$ ,  $\bar{\alpha}_f = \bar{k}_f = 0.65$ . The desired trajectories are  $\phi_d = \frac{\pi}{2} \sin(2\pi t) + \pi$  and  $\ell_d = 0.2 \sin(2\pi t) + 1$ . The true plant parameters are assumed to be  $\alpha_m = k_m = 0.35$ ,  $\alpha_s = k_s = 0.85$ ,  $\alpha_f = k_f = 0.45$  for all the simulation cases. In disturbed cases, we also assume a disturbances  $20\sin(\pi t) + 25\sin(2\pi t)$  acts on both axes.

#### 5.1 Adaptive Observer Control

The simulation results of the nominal case are shown in Fig. 3. It can be seen that it can follow the desired trajectory well. Figure 4 shows the results of the disturbed case. The adaptive observer fails to drive the system to the desired trajectory. This is because that when unknown external disturbances exist, the estimated parameter values deviate from the true values and thus the control signal is improper.

#### 5.2 Sliding Mode Control

The control parameters are  $\lambda_1 = 15$ ,  $\lambda_2 = 8$ ,  $\phi_\phi = 0.05$ , and  $\phi_\ell = 0.02$  for the following simulations. The simulation results of the disturbed case are shown in Fig. 5 and 6. We can see that the tracking performance is adequate. However, the control signal chatters due to the high feedback gains.

#### 5.3 Disturbance Estimation Based Tracking Control

The control gains  $K_o = [62.083 \ 8.11]$ , and

$$P = \begin{bmatrix} 129.375 & 0.4687 \\ 0.4687 & 1.9043 \end{bmatrix}$$

are used in the simulations. The simulation results of the disturbed case are shown in Fig. 7 and 8. We can see that the control algorithm works well in both cases. Compare the tracking error of Fig. 6 with Fig. 8, we can see that the control signal as well as tracking error of the disturbance based control are smaller compared to the previous two algorithms.

### 6. Conclusions

A disturbance estimation based tracking control scheme for nonlinear systems has been proposed. A telescopic robot manipulator is used as an application example. The proposed tracking control algorithm requires little knowledge of system structures, since the system uncertainty, unmodeled dynamics and

external disturbances are lumped as the overall disturbance. Based on the proposed disturbance estimation scheme, a tracking controller is constructed which is asymptotically stabilizing in the sense of Lyapunov. The disturbance estimation based tracking control is compared with a classical adaptive controller and a sliding mode controller. The proposed control algorithm was found to generate superior tracking performance and smoother control action. This superior performance of the proposed method is due to the fact that all the system nonlinearities, uncertainties, unmodeled dynamics, and external disturbances are compensated by the disturbance estimation without requiring large feedback gains.

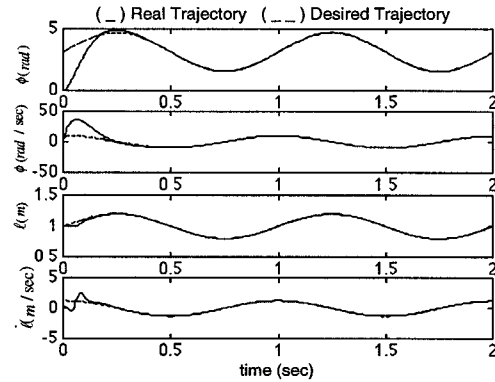


Fig. 3 Adaptive control (nominal case)

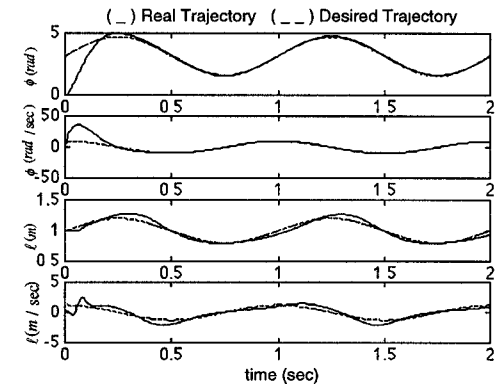


Fig. 4 Adaptive control (disturbed case)

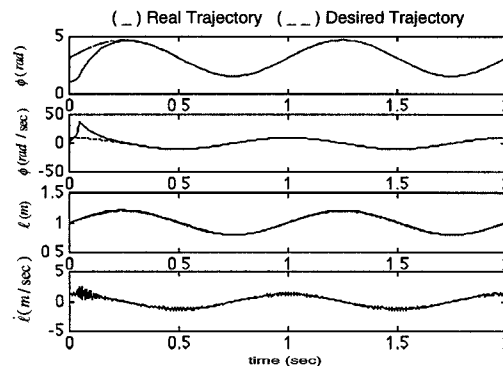


Fig. 5 Sliding mode control (disturbed case)

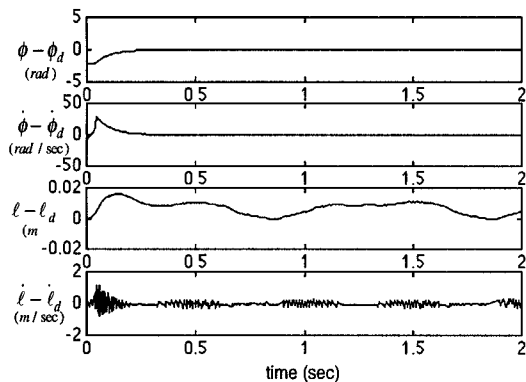


Fig. 6 Sliding mode control (disturbed case)

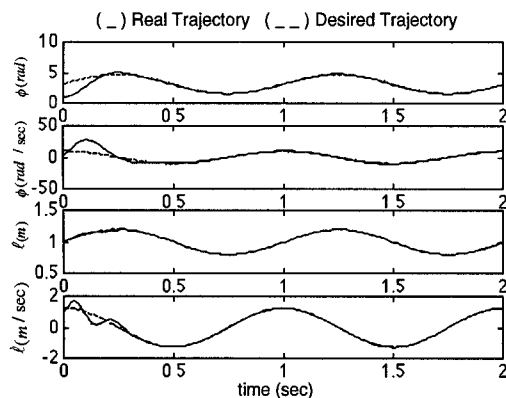


Fig. 7 Disturbance estimation based control (disturbed case)

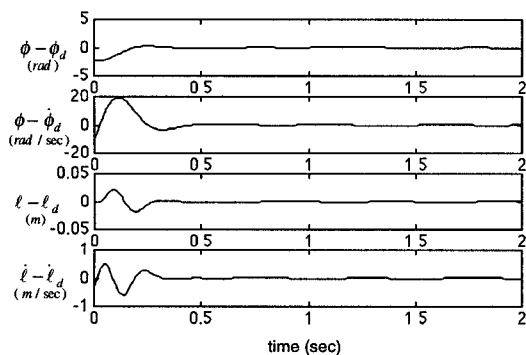


Fig. 8 Disturbance estimation based control (disturbed case)

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