LQ and H_∞ Preview Control for a Durability Simulator

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Abstract

The tracking control of a suspension durability test simulator is investigated in this paper. The objective is to control a hydraulically actuated durability simulator, so that vehicle responses, previously measured on the test track, can be reproduced in the laboratory. A two-degree-of-freedom (2DOF) H_∞ control design procedure is proposed to improve tracking performance in the face of noise and disturbances. The 2DOF formulation consists of a feedback and feedforward term, which is directly extended to include preview control.

Nomenclature

b	feedback control signal
c_s	suspension damping
c_t	tire damping
e	error between desired and achieved response
f	feedforward control signal
F	feedforward controller
G	nominal plant model
K	feedback controller
k_s	suspension stiffness
k_t	tire stiffness
S	Sensitivity function
T	Complimentary Sensitivity function
m_s	sprung mass
m_u	unsprung mass
n	measurement noise
x_{act}	actuator displacement
x_u	unspung mass
x_s	sprung mass
y_d	desired response of the plant
y	achieved response of the plant
W_b	weighting on the control signal
W_d	weighting on the disturbance
W_e	weighting on the error
W_n	weighting on the noise signal

1 Introduction

The use of laboratory road simulators has become quite common in the Automotive Industry. Road simulators have significantly reduced vehicle development time by moving testing that was formerly done on the test track, into the laboratory. One particularly important application of the road simulator is in assessing suspension durability for vehicle development and design verification. These durability simulators are used to reproduce vehicle responses (e.g. unsprung mass acceleration measured on the test track) in the laboratory. Once the obtained vehicle responses have been shown to be correlated with the test track, they can be repeated a specified number of times to determine useful life of the suspension components.

Techniques for control of durability simulators have been documented in [1-2]. The current, industry-standard approaches to control all rely on the use of simple, experimentally determined models of the simulator and vehicle dynamics. These models are used to predict the control inputs necessary to achieve the desired responses. Although the desired vehicle responses are know apriori, an open-loop, iterative control process is used. The iterative control process is very time-consuming and does not compensate for changes in the vehicle parameters due to design evolution or suspension degradation as the durability test proceeds.

It is well-known that feedforward control can improve tracking performance without deteriorating closed loop robustness. The 2DOF H_∞ formulation has the benefit of an extra DOF to improve tracking performance in the face of noise, disturbances, and modeling uncertainty. These can be due to nonlinearities, unmodeled dynamics, and changes in plant parameters. This is accomplished by reducing the loop bandwidth for better noise attenuation and disturbance rejection and increasing the transmission bandwidth, using simultaneous synthesis of the pre-filter, for improved tracking performance.

2DOF H_∞ control approaches have been documented in [3-4]. However, these approaches are somewhat complicated and can lead to higher controller order. These

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factors can make implementation more difficult. In this paper, a simple approach is described which designs the feedback and feedforward terms simultaneously and maintains the controller order of a SDOF approach.

In the case where future desired outputs are known, preview control techniques can also be used to further enhance tracking performance. There are at least two LQ preview control formulations that are quite well known. In [5], the proposed control consists of one feedback and two feedforward terms. The feedback is the standard LQ result. The feedforward terms involve the preview signal inside the preview window (a convolution term) and outside the preview window (a "kick" term). If the "kick" feedforward term is neglected, as in [6], a much simpler preview control algorithm is obtained. Both algorithms have been implemented and favorable test results have been reported.

Recently, H_∞ preview designs have gained increasing interest [7]. This approach is based on game theory, and the Riccati Equation is modified accordingly. The feedforward control law is assumed unchanged. In this paper, we propose a different design approach. The simultaneous 2DOF design is extended to include an H_∞ preview that is analogous to well-know LQ derivations.

The obtained H_∞ preview derivation will be compared to the classical LQ preview using minimum principle derivations.

2.1 Standard SDOF H_∞ Control

A typical block diagram for a standard SDOF H_∞ characterization is shown in Figure 1. Frequency-dependent weighting functions are used to characterize noise and disturbances as well as constrain the control energy and tracking performance errors.

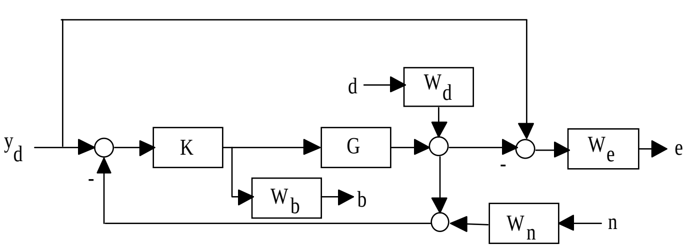

Figure 1. SDOF H_∞ Problem with Noise and Disturbance

For the system shown in Figure 1, it is desired to minimize the infinity norm for the transfer function from y_d, d, and n to the outputs e and b. The transfer functions are obtained as follows

$$\tilde{e} = y_d - y$$

$$\tilde{e} = y_d - W_d d + GKW_n n - GK(y_d - y)$$

$$\tilde{e} = S(y_d - W_d d + GKW_n n)$$

Where S is the Sensitivity Function defined as

$$S \equiv (I + GK)^{-1}$$

$$\tilde{e} = Sy_d - SW_d d + TW_n n$$

$$e = W_e \tilde{e}$$

so

$$e = W_e S \cdot y_d - W_e SW_d d + W_e TW_n n$$

Now

$$\tilde{b} = K \cdot error$$

where

$$error = S(y_d - W_d d - W_n n)$$

$$b = W_b \tilde{b}$$

$$b = W_b KSy_d - W_b KSW_d d - W_b KSW_n n$$

and

$$\begin{bmatrix} e \\ b \end{bmatrix} = \begin{bmatrix} W_e S & -W_e SW_d & W_e TW_n \\ W_b KS & -W_b KSW_d & -W_b KSW_n \end{bmatrix} \begin{bmatrix} y_d \\ d \\ n \end{bmatrix}$$

To solve the H_∞ problem, transform the state space representation of Figure 1, into the standard 2-port problem with augmented plant, P, shown in Figure 2, where the object is to find a stabilizing controller, *K*, to minimize the transfer function from y_d, d, and n to e and b.

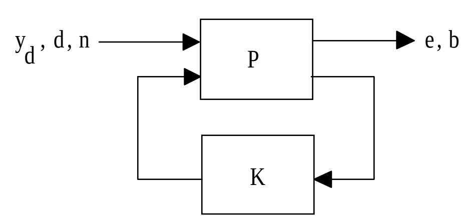

Figure 2. Standard 2-Port Problem

2.2 2DOF H_∞ Control

A 2DOF H_∞ characterization is shown in Figure 3. Frequency-dependent weighting functions are again used to characterize the inputs to the system and to constrain the control energy and tracking performance errors.

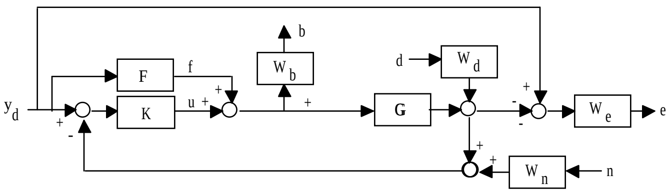

Figure 3. Two-Degree-of-Freedom Tracking Controller

For the system shown, it is desired to minimize the infinity norm of the transfer function from the inputs y_d, d, and n to the outputs b and e. In order to design the feedback and feedforward controllers simultaneously, the problem can be reformulated with a coupled two-degree-of-freedom controller as shown in Figure 4. Note that in order to satisfy the rank conditions of the standard H_∞ control problem a weight W_f was added. The value of W_f was set to a small number that is determined through trial and error.

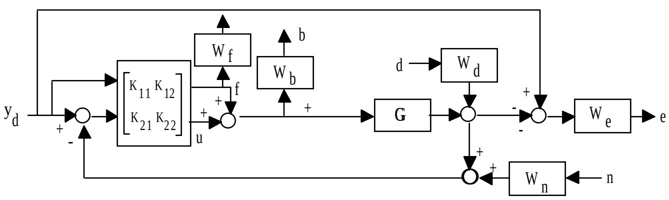

Figure 4. Two-Degree-of-Freedom Coupled Tracking Controller

The outputs of the coupled controller are

$$\tilde{f} = K_{11} y_d + K_{12} E \quad \text{and} \quad u = K_{21} y_d + K_{22} E$$

If required, the coupled H_∞ controller that is obtained can be decoupled because

$$\tilde{b} = (K_{11} + K_{21}) y_d + (K_{12} + K_{22}) E$$

$$\tilde{b} = F y_d + KE$$

where

$$F \equiv (K_{11} + K_{21})$$

and

$$K \equiv (K_{12} + K_{22})$$

However, a separate, uncoupled feedback and feedforward controller requires a larger number of states than the single coupled controller. The transfer functions to be minimized can be obtained as in the previous section and are shown below

$$\begin{bmatrix} e \\ b \end{bmatrix} = \begin{bmatrix} W_e S(I - GF) & -W_e S W_d & W_e T W_n \\ W_b KS(I - GF) & -W_b KSW_d & -W_b KSW_n \end{bmatrix} \begin{bmatrix} y_d \\ d \\ n \end{bmatrix}$$

2.3 LQ Preview Algortithm

There a several ways to approach the LQ preview problem. The approach chosen here is Pontriagin's minimum principle, which is stated with the following system and resulting Hamiltonian

$$\dot{x} = Ax + Bu + B_p p$$
$$y = Cx$$

where p is the previewable disturbance

$$H(x, \lambda, t) = \frac{1}{2}\left[x^T Q x + u^T R u\right] + \lambda^T (Ax + Bu + B_p p)$$

$$\frac{\partial H}{\partial u} = 0 = Ru + B^T \lambda$$
$$u = -R^{-1} B^T \lambda$$

$$\dot{x} = \frac{\partial H}{\partial \lambda} = Ax + Bu + B_p p$$
$$\dot{x} = Ax - BR^{-1} B^T \lambda + B_p p$$

$$-\dot{\lambda} = \frac{\partial H}{\partial x} = A^T \lambda + Qx$$
$$\dot{\lambda} = -Qx - A^T \lambda$$

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A & -B\,R^{-1} B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} + \begin{bmatrix} B_p p \\ 0 \end{bmatrix}$$

Assuming a solution for λ.

$$\lambda = Px + \int_0^{t_{la}} F_1(t,\tau) p(t+\tau) d\tau + F_2(t) p(t + t_{la})$$

which is differentiated on both sides w.r.t to time.

$$\frac{d\lambda}{dt} = \frac{dP}{dt} x + P \frac{dx}{dt} + \int_0^{t_{la}} F_1(t,\tau) \frac{d}{dt} p(t+\tau) d\tau +$$
$$\int_0^{t_{la}} \frac{d}{dt} F_1(t,\tau) p(t+\tau) d\tau + \dot{F}_2(t) p(t+t_{la}) + F_2(t) \dot{p}(t+t_{la}) =$$
$$-Qx - A^T (Px + \int_0^{t_{la}} F_1(t,\tau) p(t+\tau) d\tau + F_2(t) p(t+t_{la}))$$

substituting into the previous equation and grouping terms

x terms:

$$\frac{dP}{dt} + PA - PBR^{-1} B^T P + A^T P + Q = 0$$

which is the standard LQ Riccati equation integral terms:

$$-PBR^{-1}B^T F_1(t,\tau) - \frac{d}{d\tau}F_1(t,\tau) + \frac{d}{dt}F_1(t,\tau) + A^T F_1(t,\tau) = 0$$

$p(t)$ terms:

$$F_1(t,0) = PB_p$$

$p(t+t_{la})$ terms:

$$-PBR^{-1}B^T F_2(t) + F_1(t,t_{la}) + \dot{F}_2(t) + F_2(t)A_w + A^T F_2(t) = 0$$

where $\frac{dp}{dt} \equiv A_w p$

from which the feedback and feedforward gains are solved

2.4 H$_\infty$ Preview Algorithm

The approach to the H$_\infty$ preview problem is directly analogous to the LQ case.

$$H(x,\lambda,t) = \frac{1}{2}\left[x^T Qx + u^T Ru - \gamma^2 w^T w\right] + \lambda^T (Ax + B_1 w + B_2 u + B_p p)$$

$$\frac{\partial H}{\partial u} = 0 = Ru + B_2^T \lambda$$

$$u = -R^{-1}B_2^T \lambda$$

$$\frac{\partial H}{\partial w} = 0 = -\gamma^2 w + B_1^T \lambda$$

$$w = \gamma^{-2}B_1^T \lambda$$

$$\dot{x} = \frac{\partial H}{\partial \lambda} = Ax + B_1 w + B_2 u + B_p p$$

$$\dot{x} = Ax + \gamma^{-2}B_1 B_1^T \lambda - B_2 B_2^T \lambda + B_p p$$

$$-\dot{\lambda} = \frac{\partial H}{\partial x} = A^T \lambda + Qx$$

$$\dot{\lambda} = -A^T \lambda - Q_1 x$$

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A & \gamma^{-2}B_1 B_1^T - B_2 B_2^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} + \begin{bmatrix} B_p p \\ 0 \end{bmatrix}$$

Assuming a solution for λ

$$\lambda = Px + \int_0^{t_{la}} F_1(\tau)B_p p(t+\tau)d\tau + F_2(t)p(t+t_{la})$$

and differentiating both sides w.r.t. time

$$\frac{d\lambda}{dt} = \frac{dP}{dt}x + P\frac{dx}{dt} + \int_0^{t_{la}} F_1(\tau)B_p \frac{d}{dt}p(t+\tau)d\tau$$
$$+\dot{F}_2(t)p(t+t_{la}) + F_2(t)\dot{p}(t+t_{la}) = -Qx$$
$$-A^T(Px + \int_0^{t_{la}} F_1(\tau)B_p p(t+\tau) + F_2(t)p(t+t_{la}))d\tau$$

Substituting into the above equation for x and λ and grouping terms

x terms:

$$\frac{dP}{dt} + PA + \gamma^{-2}PB_1B_1^T P - PB_2B_2^T P + A^T P + Q = 0$$

which is the H$_\infty$ Riccati equation

integral terms:

$$\gamma^{-2}PB_1B_1^T F_1(\tau) - PB_2B_2^T F_1(\tau) + \frac{d}{d\tau}F_1(\tau) + A^T F_1(\tau) = 0$$

$p(t)$ terms:

$$F_1(0) = PB_p$$

$p(t+t_{la})$ terms:

$$\gamma^{-2}PB_1B_1^T F_2(t) - PB_2B_2^T F_2(t) + F_1(t_{la}) + \dot{F}_2(t)$$
$$+F_2(t)A_w + A^T F_2(t) = 0$$

3.1 System Description

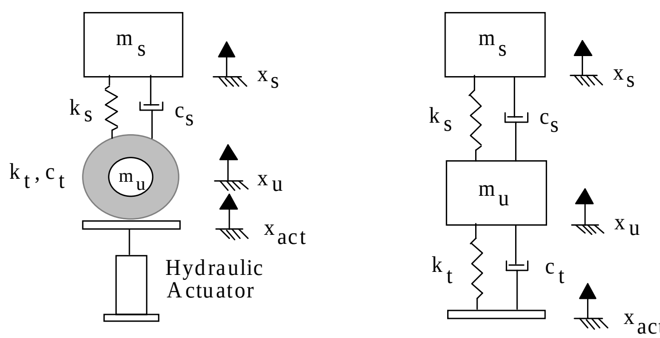

Figure 5. Vehicle Durability Simulator Schematic

The schematic of a vehicle durability simulator is shown in Figure 5. The control input is the actuator displacement, x_{act}, which provides the road input into the system. The desired response to be tracked is the unsprung mass acceleration. If the states are chosen as

$$[x] = \begin{bmatrix} x_s - x_u \\ \dot{x}_s \\ x_u - x_{act} \\ \dot{x}_u \end{bmatrix}$$

Then, the system can be represented as

$$[\dot{x}] = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -k_s/m_s & -c_s/m_s & 0 & c_s/m_s \\ 0 & 0 & 0 & 1 \\ k_s/m_u & c_s/m_u & -k_t/m_u & -(c_t+c_s)/m_u \end{bmatrix} [x] + \begin{bmatrix} 0 \\ 0 \\ -1 \\ c_t/m_u \end{bmatrix} \dot{x}_{act}$$

$$y = \left[k_s/m_u \quad k_s/m_u \quad -k_t/m_u \quad -(c_t + c_s)/m_u \right][x] + \left[c_t/m_u \right]\dot{x}_{act}$$

3.2 System Identification

The system model is obtained using system identification methodology from experimental input/output data. The excitation signal is a broad-band (0-50 Hertz)input to the durability simulator and the unsprung mass acceleration response is collected. A model, which includes actuator dynamics, was then obtained between the actuator input and the unsprung mass acceleration. It was found that the system could be accurately represented for controller design with a ARX model that was converted to a state-space realization.

4 Results

The simulation results for the SDOF H_∞ controller are shown in Figure 6. Frequency dependent weighting functions were used to represent noise and disurbance entering the system. As can be seen, the tracking performance suffers at the expense of attenuating the noise and disturbances to the system

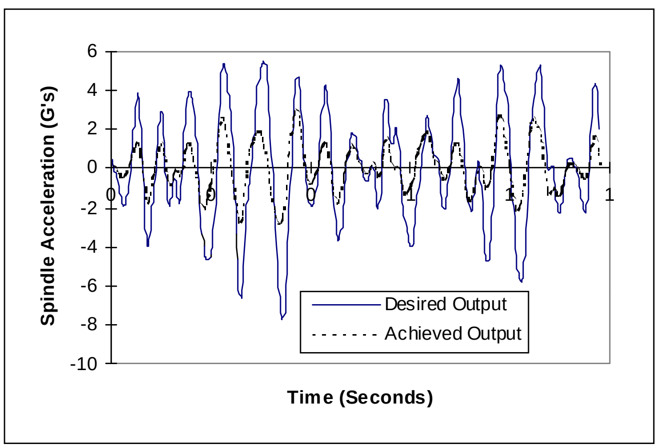

Figure 6. SDOF H_∞ Tracking Performance

As of the completion of this paper, the analytical algorithms for the H_∞ preview had not been implemented in the simulations. Instead, a numerical preview approach was used. This is accomplished by modifying the augmented plant by inserting delays into the feedback signal. The same noise and disturbance weights as well as performance and control constraints were used as in the SDOF case. The performance of the 2DOF H_∞ preview controller is shown in Figure 7. It can be seen that the performance is far superior to the SDOF in the face of noise and disturbance, due to the presence of the feedforward term.

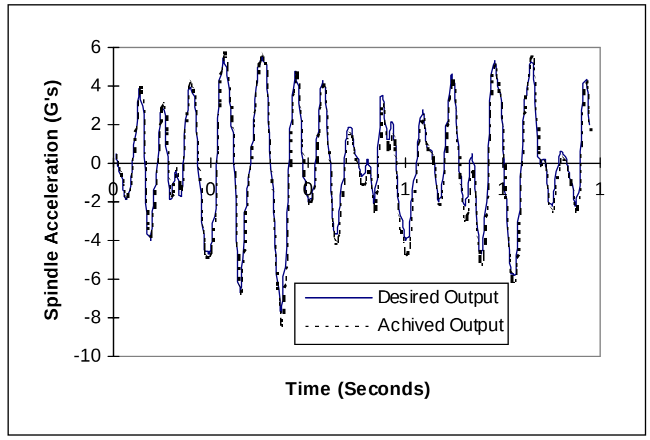

Figure 7. 2DOF H_∞ Preview Tracking Performance

5 Conclusions

A method of obtaining a 2DOF H_∞ control has been demonstrated. The resulting control is conceptually simple and has the same order as a SDOF controller. The 2DOF approach was directly extended to an H_∞ preview law. The development was compared to classical LQ preview and shown to be analogous. A SDOF feedback-only and a numerically implemented 2DOF H_∞ preview controller for a durability simulator were compared through simulations. The H_∞ preview controller was shown to give far better performance in the face of noise and disturbance.

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