A Novel Active Suspension Design Technique--Simulation and Experimental Results

Huei Peng Ryan Strathearn A. Galip Ulsoy
Department of Mechanical Engineering
and Applied Mechanics
University of Michigan
Ann Arbor, MI 48109-2125
hpeng@umich.edu
Tel: (313) 936-0352
Fax:(313) 647-3170

Abstract

A novel approach for the design of active suspension systems is proposed in this paper. By defining *virtual input* signals, the dynamics of a quarter-car suspension system is transformed and becomes independent of vehicle parameters. Standard linear quadratic (LQ) optimization technique is applied to design the virtual inputs. The virtual input signal is used to construct the desired trajectory of a subloop control system. The subloop control problem is designed using classical SISO control design techniques plus an ad hoc preview enhancement. It was shown that this virtual input based (VIB) approach results in a simple outer loop design because it is independent of system parameters. Simulation as well as experimental results are reported.

1. Introduction

Vehicle suspension systems have developed over the last 100 years to a very high level of sophistication. Most manufacturers today use a passive suspension system employing some type of springs in combination with hydraulic or pneumatic shock absorbers. Despite the wide range of designs currently available, passive suspensions, because they can only store and dissipate energy in a pre-determined manner, will always be a compromise between passenger ride comfort, handling, and suspension stroke over the operating range.

Active suspensions have been extensively studied in the last 30 years (e.g. [1]-[3]). Most of the past work uses the quarter-car model, which includes only two degree-of-freedom of the vehicle motion in the vertical direction. In many cases, the active force between the sprung mass and unsprung mass is assumed to be controlled directly. The control algorithms are usually based on Linear Quadratic (LQ), H_∞ Fuzzy, or LQ based preview control approaches. In the computer simulations, it was shown that active suspension can improve all the three performance aspects (passenger ride comfort, handling, and rattle space) compared with passive designs at a moderate cost (control effort, hardware complexity and cost). It is fair to say that the un-constrained active suspension design process is well established, and the performance trade-off issues are well understood.

Practical (implementation) issues such as observer designs, actuator power and range constraints and power consumption, etc. have also been widely studied. Observers are necessary for LQ and other full-state feedback control algorithms since suspension stroke is frequently the only feedback signal available. The other state variables need to be estimated accordingly. When vehicle parameters change, the resulted LQG (LQ control based on state observers) control algorithms may become unstable, unless conservative observers are used. Design of the observer thus is usually done through an iterative process.

Most of the past active suspension designs were developed based on the quarter-car model (Figure 1). The following state-space model can be easily obtained from Figure 1:

$$\frac{d}{dt}\begin{bmatrix} z_{us}-z_0 \\ \dot{z}_{us} \\ z_s-z_{us} \\ \dot{z}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{us}}{m_{us}} & -\frac{(c_s+c_{us})}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} \end{bmatrix} \begin{bmatrix} z_{us}-z_0 \\ \dot{z}_{us} \\ z_s-z_{us} \\ \dot{z}_s \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{m_s}{m_{us}} \\ 0 \\ 1 \end{bmatrix} \frac{f}{m_s} + \begin{bmatrix} -1 \\ \frac{c_{us}}{m_{us}} \\ 0 \\ 0 \end{bmatrix} \dot{z}_0 \quad (1)$$

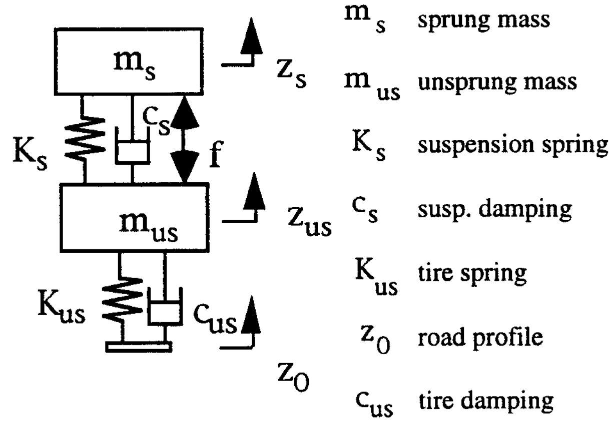

Figure 1 Schematic diagram of the quarter-car model

where the active control force f is usually assumed to be directly manipulable, and a subloop controller is

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available to drive the actuator force to approach the desired value quickly. Although this approach is common, there are essentially two drawbacks: (1) The design of the subloop control algorithm is not addressed, and eventually this issue will surface and may require re-design of the main loop control or observer algorithms; and (2) If the subloop is openloop, the actual force is not guaranteed to track the desired value. If it is closed-loop, a force sensor maybe required.

To address the above drawbacks and to improve overall system performance, several researchers have proposed to include the actuator dynamics to form an enhanced plant model (e.g. [4]). The new control input thus becomes an electrical signal that could be manipulated much more quickly and precisely and therefore eliminates the need for a subloop design. For example, when a hydraulic actuator is used, the following equations can be obtained by linearizing the actuator dynamics [4]:

$$2C_x x_{sv} + 2A_p(\dot{z}_{us} - \dot{z}_s) = \frac{V}{\beta A_p}\dot{f} \qquad (2)$$

$$\tau \dot{x}_{sv} + x_{sv} = K_v i_{sv} \qquad (3)$$

where C_x is the flow gain for spool valve, x_{sv} is the spool valve displacement, A_p is the area of the actuator piston, V is the hydraulic cylinder volume in one travel direction, β is the effective bulk modulus of hydraulic fluid, and f is the actuator force. The spool valve is modeled as a first-order lag system with time constant τ. The input to the spool valve is the valve control current i_{sv}, and K_v is the valve proportional constant. Notice that Eq.(2) is slightly different from that originally presented in [4] since the operating point is selected differently. Combining Eqs.(1)-(3), the state space equation of the enhanced plant (suspension plus hydraulic actuator) is

$$\frac{d}{dt}\begin{bmatrix} z_{us}-z_0 \\ \dot{z}_{us} \\ z_s-z_{us} \\ \dot{z}_s \\ f \\ x_{sv} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{k_{us}}{m_{us}} & -\frac{c_s+c_{us}}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} & \frac{-1}{m_{us}} & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{1}{m_s} & 0 \\ 0 & \frac{2\beta A_p^2}{V} & 0 & -\frac{2\beta A_p^2}{V} & 0 & \frac{2C_x\beta A_p}{V} \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{\tau} \end{bmatrix}\begin{bmatrix} z_{us}-z_0 \\ \dot{z}_{us} \\ z_s-z_{us} \\ \dot{z}_s \\ f \\ x_{sv} \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{K_v}{\tau} \end{bmatrix} i_{sv} + \begin{bmatrix} -1 \\ \frac{c_{us}}{m_{us}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\dot{z}_0 \qquad (4)$$

$$\equiv A_{ad}\underline{x}_{ad} + B_{ad}\underline{u}_{ad} + G_{ad}w$$

Classical control design approaches (e.g. LQG) can then be applied to design active suspension algorithms. One of the obvious drawbacks is that the resulting control algorithm requires full-state feedback, or a competent observer. Due to the higher system order, mixed of slow (suspension) and fast (servo valve) dynamics, and the fact that the observability (output assumed to be the suspension stroke) is a lot inferior, the implementation of the LQG algorithm can be even more difficult on the enhanced plant.

2. The Virtual Input Based (VIB) Approach

In this section, a different approach is proposed to circumvent the difficulties in the control design presented in the previous section. Assuming that the active control signals are the sprung and unsprung mass accelerations ($\ddot{z}_{us}$ and $\ddot{z}_s$), the suspension dynamics become

$$\frac{d}{dt}\begin{bmatrix} z_{us}-z_0 \\ \dot{z}_{us} \\ z_s-z_{us} \\ \dot{z}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} z_{us}-z_0 \\ \dot{z}_{us} \\ z_s-z_{us} \\ \dot{z}_s \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \ddot{z}_{us} \\ \ddot{z}_s \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\dot{z}_0$$

$$\equiv A\underline{x} + B\underline{u} + Gw \qquad (5)$$

Notice that all entries of the system and input matrices are constant and do not depend on the vehicle parameters. Eq.(5) simply states the physical relationship between the virtual inputs and the road input to the states. Notice that the state variables of the new system are selected to be exactly the same as those of the traditional quarter-car suspension. One of the most praised feature of the LQ based suspension design is that it offers a natural trade-off tuning among the various suspension performance objectives. In this study, we will use the LQ design approach to compute the virtual control signal $\ddot{z}_{us}$ and $\ddot{z}_s$ for the system shown in Eq.(5). The cost function to be minimized is selected to be

$$J = \frac{1}{2}\int\left[r_1(z_{us}-z_o)^2 + r_2(z_s-z_{us})^2 + \ddot{z}_s^{\,2} + \rho\ddot{z}_{us}^{\,2}\right]dt$$

$$= \frac{1}{2}\int\left[\underline{x}^T Q\underline{x} + \underline{u}^T R\underline{u}\right]dt$$

$$= \frac{1}{2}\int \underline{x}^T\begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & r_2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\underline{x} + \underline{u}^T\begin{bmatrix} \rho & 0 \\ 0 & 1 \end{bmatrix}\underline{u}dt \qquad (6)$$

One can then simply tune the magnitude of r_1, r_2 and ρ to change the (desired) relative importance of tire deflection (road holding), suspension stroke, ride quality, and axle durability to obtain the active suspension control laws . The proper virtual control signal to achieve this desired trade-off will then have the following full-state feedback form:

$$\underline{u} = \begin{bmatrix} \ddot{z}_{us} \\ \ddot{z}_s \end{bmatrix} = -K\underline{x} = -\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \begin{bmatrix} z_{us} - z_0 \\ \dot{z}_{us} \\ z_s - z_{us} \\ \dot{z}_s \end{bmatrix} \qquad (7)$$

where the feedback gain K is the solution of the Algebraic Riccati Equation (ARE) corresponding to the (A,B,Q,R) matrices described in Eqs.(5) and (6). K_1 and K_2 are 1×4 row vectors to compute the virtual inputs $\ddot{z}_{us}$ and $\ddot{z}_s$.

The overall suspension control design problem has now been reduced to the subloop control problem shown in Figure 2. The objective is to find the subloop control G_c so that it stabilizes the overall system, and achieves good suspension performance even when the vehicle parameters vary.

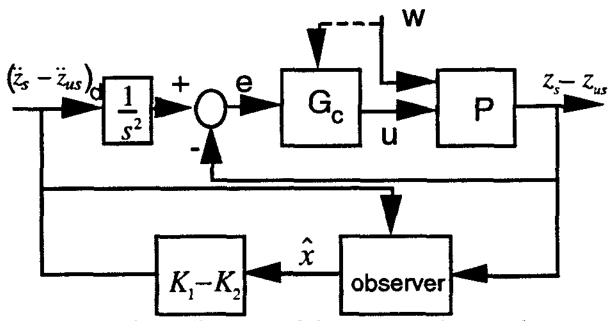

Figure 2 Block diagram of the proposed control system

The plant P shown in Figure 2 could be based on an enhanced suspension model that includes actuator dynamics. Therefore, the control signal u can be manipulated quickly. The controller G_c, however, uses only an error signal between the desired suspension stroke and the true suspension stroke and possibly the road disturbance for preview compensation. Since the main-loop feedback gain $(K_1 - K_2)$ is obtained from the desired performance trade-off represented in the cost function, It is fair to say that the main difference between this proposed active suspension structure and previous designs is that we have decomposed the control design into two parts: a trajectory planning part, and a regulation (tracking) part. The desired trajectory is generated by considering the trade-off between the three suspension objectives, and the subloop is design to track this desired trajectory as close as possible. It should be noted that the estimated state vector $\hat{\underline{x}}$ in Figure 2 could be generated from either the 4th order system or the 6th order system. The observer based on the 4th order system can be easily designed to guarantee BIBO stable, since the state, input and output matrices are all constant. However, we will need both suspension and tire deflection measurements, which could be hard to get. Alternatively, the states can be estimated based on the 6th order plant, and we can focus on the estimation of the first four states, since the augmented states are not necessary for the calculation of the virtual inputs.

3. Subloop Control Design

The design of the subloop control law C can take advantage of the abundance of classical tracking control design techniques. For example, Due to the fact two integrators have been used, in the main loop, a simple proportional or lead/lag control was found to be adequate to stabilize the overall system. Standard robust control techniques can also be applied. When future road disturbance can be previewed, the preview control algorithms can be used to further improve the disturbance rejection performance. In the following, a simplified system (Figure 3) will be used for the control design and analysis purposes, which ignores the observer dynamics.

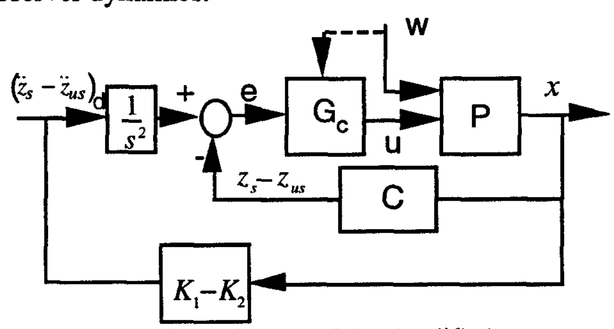

Figure 3 Block diagram of the simplified system

For its simplicity and satisfactory performance, a simple P-control algorithm is selected for the subloop feedback control. After careful study of the root locus of the overall closed-loop system under a range of sprung mass values, it was determined that K_p=0.002 works satisfactorily under all possible vehicle load variations.

To improve the overall disturbance rejection performance, however, we suggest to use the LQ-preview enhanced control law, which has the following form:

$$u(t) = u_b + u_f = K_p e(t) + \int_0^{t_{la}} F_1(\tau) w(t+\tau) d\tau \qquad (8)$$

where t_{la} is the preview time. It should be noted that the preview control part, which is determined by the preview gain F_1 is the optimal preview control gain in the sense of LQ (see [5][6] for details). Here we have borrowed it to work with the proportional feedback control algorithm in an ad hoc manner. This approach, however, has been applied to several experiments (e.g. [7]) and the results are usually quite favorable.

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To design the LQ preview control gains, we need to apply the preview control theory to the augmented state equation which has eight state variables:

$$\frac{d}{dt}\begin{bmatrix}\underline{x}_{ad}\\ r\\ \dot{r}\end{bmatrix} = \begin{bmatrix}A_{ad}-B_{ad}K_pC & B_{ad}K_p & 0\\ 0 & 0 & 1\\ K_1-K_2 & 0 & 0\end{bmatrix}\begin{bmatrix}\underline{x}_{ad}\\ r\\ \dot{r}\end{bmatrix} + \begin{bmatrix}B_{ad}\\ 0\\ 0\end{bmatrix}u_f + \begin{bmatrix}G_{ad}\\ 0\\ 0\end{bmatrix}\dot{z}_0$$

$$\equiv A_e\underline{x}_e + B_e u_f + G_e w \qquad (9)$$

where the 6-state system shown in Eq.(4) is augmented with the two states corresponding to the two integrators. The variable *r(t)* denotes the output from the double integrator, which is also the reference signal to the subloop. Base on Eq.(9), we can choose proper LQ weighting matrices Q_e and R_e, and the infinite-horizon preview gains can be obtained[7]:

$$F_1(\tau) = e^{(A_e - B_e R_e^{-1} B_e^T K_{ss})^T \tau} K_{ss} G_e \qquad (10)$$

where K_{ss} is the solution of the algebraic Riccati Equation (ARE) for the given LQ optimization problem (specified A_e, B_e, Q_e and R_e).

4. Simulation and Experimental Results

The suspension and hydraulic system parameters used in the simulations are taken from the University of Michigan quarter-car test rig made available by the Ford Motor Company. This test rig uses a pneumatic suspension spring, and a hydraulic active suspension actuator. The road actuator is also hydraulic, and is of similar capacity. The parameters (see Eq.(4)) are summarized in Table 1.

Table 1 Suspension and hydraulic parameters

Var.	Meaning	Value	Var.	Meaning	Value
m_s	sprung mass	253kg	τ	valve time constant.	0.03sec
m_{us}	unsprung mass	26kg	K_v	Valve gain	1.73 m/Amp
C_s	suspension damping	1500 N/m/sec	A_p	cylinder area	0.0011 m^2
C_{us}	tire damping	10N/m/sec	C_x	valve flow gain	0.923 m^2/sec
K_s	suspension stiffness	12000 N/m	V	cylinder volume	1.1e-4 m^3
K_{us}	tire stiffness	90000 N/m	β	bulk modulus	1.6e6 N/m^2

The simulation results comparing the proposed Virtual-Input-Based (VIB) LQ-mainloop plus simple proportional-subloop control is compared against an LQ control (designed in the traditional way). The response from the passive suspension is also presented for reference. It can be seen that the performance improvement from the VIB and LQ controls are similar. The LQ gains are designed to be slightly more conservative for the purpose of separate the graph traces. However, the VIB control force was found to be much smoother because of the double integrator in the main loop. In this simulation, no measurement noises are used. The road excitation is essentially a 1Hz sinusoidal signal, and a very mild white noise is added on top of it. We except that VIB also has superior noise rejection characteristic because of the double integrator.

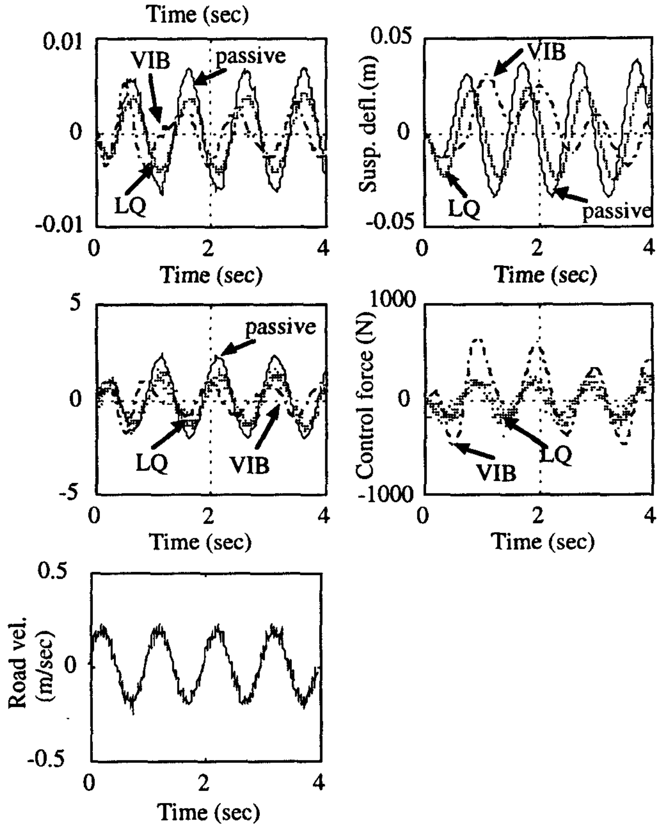

Figure 4 Comparison between the VIB, LQ and passive suspension systems

When preview control algorithm is used, the performance of the VIB control can be further improved. Figure 5 shows the simulation results of the VIB control with and without preview enhancement under the same road excitation. The preview time is assumed to be 0.15 sec, and a very crude approximation is used (3-step summation at 50msec each) to simplify the implementation of the preview integration. It can be seen that even under this short preview time plus crude approximation, this ad-hoc preview control improves the tire deflection and ride quality significantly. The control force is only slightly increased in magnitude and advanced in phase. The improved performance, however, is obtained at the price of noticeable increase of the suspension stroke.

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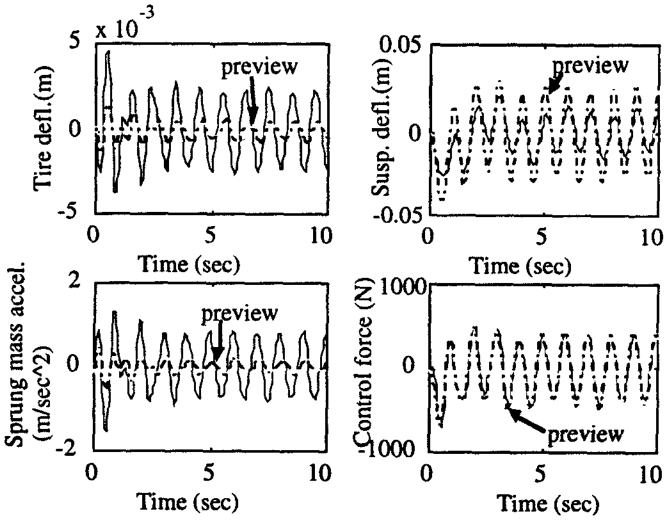

Figure 5 Simulation results of the VIB control with and without preview enhancement

From Figure 5, we can also see one of the major drawbacks of the VIB control. Due to the double integrator, two stable but lightly damped poles always exist in the overall closed-loop system. Therefore, the vehicle response floats up and down with these lightly damped modes. This phenomenon is especially obvious in the suspension deflection signal. This fact also limits the preview gain that can be used, since preview action tends to increase suspension stroke. A simple frequency response analysis (see Figure 6) also confirms that while the VIB control improves other aspects of the suspension, the low-frequency gain for the suspension deflection is even higher than traditional LQ designs.

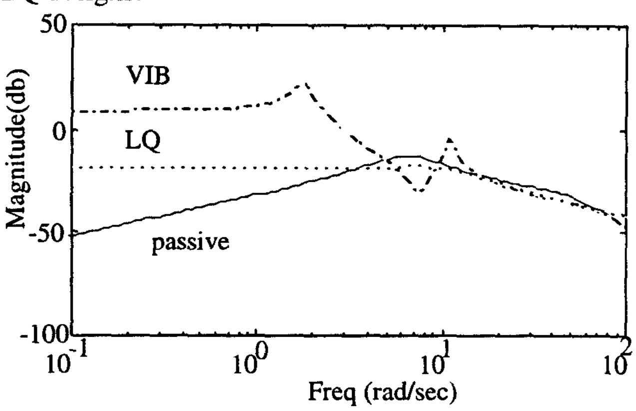

Figure 6 Frequency response of suspension deflection

In the experiments, we have added saturation limits to prevent excessive suspension deflection due to the slow modes. We have found that the suspension does float around slightly, which causes slight perturbation to tire deflection and sprung mass acceleration. However, it is within tolerable range. The control algorithm used in the experiment is the VIB feedback algorithm without the preview enhancement.

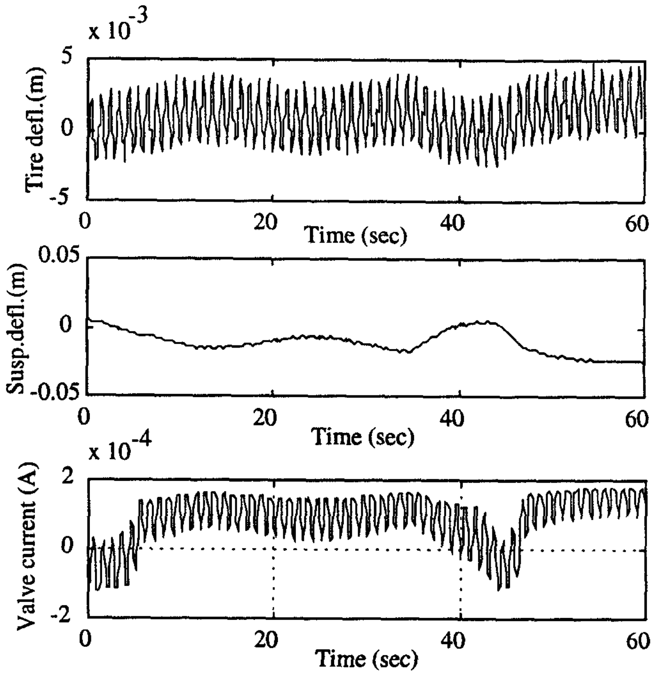

Figure 7 Experimental results of the VIB control algorithm

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