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Inverse-Dynamics Based State and Disturbance Observers for Linear Time-Invariant Systems

An output-feedback observer is proposed in this paper to simultaneously estimate unknown states and disturbances of linear time invariant systems. The states are estimated using a Luenberger-like observer while the disturbance signals are estimated based on an inverse-dynamics motivated algorithm. The proposed schemes can be applied to a wide variety of disturbances since no disturbance model is required in the estimation. Depending on the input/output rank conditions of the plant, two different designs are proposed. The observer gains are selected based on sufficient conditions for exponentially converging estimation. The design procedure is illustrated step-by-step by using two examples: a hypothetical problem and the ground vehicle lateral speed estimation problem. A standard H_∞-filter is used as the benchmark to illustrate the performance of the proposed method.
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1 Introduction

The famous Kalman Filter algorithm, which assumes white and Gaussian disturbances and noises, has been successfully applied to the estimation of state variables of linear systems in numerous engineering applications. The unknown input observer (UIO) for state estimation under unknown (non-white and/or non-Gaussian) inputs has also been intensively studied recently (e.g., [1–5]). The disturbance-decoupled observer, one of the most well known approaches, constructs a state observer by decoupling the effect of disturbances (e.g., [6–8]). Along another vein, disturbance identifiers have also been widely studied (e.g., [9,10]). The estimated disturbance is used for purposes such as improved tracking control (e.g., [11–14]) and machine health monitoring. In these studies, disturbances were frequently assumed to be the output of a known or identifiable dynamical system. In some cases, disturbances were modeled as linear combinations of predefined basis functions [10,15] or were characterized by differential equations which can be augmented into system dynamics [16], which is known in the observer literature as the "immersion" technique [17].

The methods described above focused either on state observation or disturbance estimation. Few schemes were developed to simultaneously estimate states and disturbances. One notable exception is the work by Stein and Park [18,19] where a disturbance and state estimation algorithm was derived by differentiating the output measurement. Their methods are based on the singular value decomposition concept and are applicable when a rank condition between the output matrix and the disturbance matrix is satisfied. For SISO systems, this rank condition is equivalent to the requirement that the relative degree of the transfer function is one. More recently, two other approaches have been proposed to estimate unknown inputs and states without differentiating the output measurements [20,21]. However, some norm or rank conditions are imposed on the unknown inputs in their approaches.

In this paper, we propose a state and disturbance observer algorithm for linear time invariant systems. This algorithm can be applied to a special class of nonlinear systems (see Section 2). The identification algorithms are output-feedback in nature and are based on the concept of inverse dynamics—which seems well motivated, because disturbance estimation is essentially a plant inversion problem. Since the inverse of physical systems is usually noncausal, derivatives of the output signal will help the reconstruction of the disturbance input. If the output signals are contaminated by noise, accurate output derivatives may be difficult to obtain. This has been a common problem for many output feedback disturbance observers, including the algorithms to be presented in this paper. To remedy this problem, a design parameter will be introduced, which can be adjusted to reduce the adverse effect of measurement noise.

The remainder of this paper is organized as follows. Section 2 describes the family of the systems to be studied in this paper. In Section 3, the state and disturbance observer algorithms will be presented. Sufficient conditions for exponential stability will also be shown. A standard H_∞ filter algorithm is also presented in Section 3 as the benchmark for the proposed approach. The design processes of the proposed algorithms are illustrated in Section 4. The simulation results for a real application problem-the lateral speed estimation for ground vehicles are also presented. Finally, conclusions are drawn in Section 5.

2 Problem Statement

The systems to be studied are assumed to be predominantly linear, but could include nonlinear dynamics in a special form:

$$\begin{aligned}\dot{x}(t)&=Ax(t)+\Gamma(u,y)+Bd(t)\\ y(t)&=Cx(t)\end{aligned}\tag{1}$$

where $x\in R^n$ denotes the state vector, $y\in R^{n_y}$ denotes the output vector, $u\in R^{n_u}$ is the input vector, $\Gamma(u,y)\in R^n$ is a known nonlinear function of u and y, $d\in R^{n_d}$ is the disturbance input, $A\in R^{n\times n}$ is the known state matrix, $C\in R^{n_y\times n}$ is the output matrix, and $B\in R^{n\times n_d}$ is the disturbance input matrix. It should be noted that the problem to be studied is a disturbance estimation problem rather than a control problem. Therefore, when we refer to the systems as a single-input-single-output (SISO) problem, it means $n_y=1$ and $n_d=1$ (rather than $n_u=1$). The system described in Eq. (1) is assumed to satisfy the following conditions.

Assumption 2.1:

(i) The matrices A, B and C are known. All uncertainties associated with the matrices A and B are lumped into the disturbance d.
(ii) Matrix B has full column rank.

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(iii) (A, C) are observable.

The objective of the state and disturbance estimators is to simultaneously estimate x and d from the known signals y and u. It should be noted that if the system nonlinearity is in a general form (i.e., $\Gamma(u,x)$), the unknown part of this nonlinearity need to be lumped into the disturbance term. Other than this minor change, the estimation algorithm to be presented in the following section can still be applied.

3 State and Disturbance Observer Schemes

3.1 Inverse Dynamics Based Schemes. The proposed state observer for the system given in Eq. (1) is

$$\begin{aligned}\dot{\hat{x}}(t)&=A\hat{x}(t)+\Gamma(u,y)+Bd(t)+K(y(t)-\hat{y}(t))\\ \hat{y}(t)&=C\hat{x}(t)\end{aligned} \tag{2}$$

where the symbol ˆ denotes estimation, and K is the observer gain. The error dynamics obtained from Eqs. (1) and (2) are

$$\dot{e}=A_k e+Be_d \tag{3}$$

where $A_k=A-KC$, $e_d=\hat{d}-d$ and $e=\hat{x}-x$. By examining Eq. (1), it seems reasonable to assume that the unknown disturbance $d(t)$ can be estimated from the available signals

$$\hat{d}(t)=F_1 y+F_2\dot{y}+G_1\hat{x}+G_2\dot{\hat{x}}+G_3\Gamma(u,y) \tag{4}$$

where $F_1\in R^{n_d\times n_y}$, $F_2\in R^{n_d\times n_y}$, $G_1\in R^{n_d\times n}$, $G_2\in R^{n_d\times n}$ and $G_3\in R^{n_d\times n}$ are observer gain matrices to be designed. Depending on the "invertibility" of the plant, we have two cases. The observer design for these two cases will be presented in Lemma 3.1 and 3.3, respectively. It should be noted that Eq. (4) presents a novel treatment of the disturbance estimation problem. The advantages of the proposed algorithms will be discussed at the end of Lemma 3.1, after the design process has been illustrated.

Case 1: There exists F_2, $F_2CB-I_{n_d\times n_d}=0$, where $I_{n_d\times n_d}$ denotes the $n_d\times n_d$ identity matrix.

Lemma 3.1: Assuming that the state and input observers are as described in Eqs. (2) and (4). If

(i) F_2 is selected to satisfy

$$F_2CB-I_{n_d\times n_d}=0 \tag{5}$$

(i) F_1 and K are selected such that $A-B(F_1C+F_2CA)-KC$ is Hurwitz.

Then $G_1=-(F_1C+F_2CA)$, $G_2=0$ and $G_3=-F_2C$ guarantee that the state and disturbance estimation errors e and e_d will converge exponentially to zero.

Proof: Subtracting d from Eq. (4), the disturbance estimation error was found to be

$$\begin{aligned}e_d&=F_1y+F_2\dot{y}+G_1\hat{x}+G_2\dot{\hat{x}}+G_3\Gamma(u,y)-d\\ &=(F_1C+F_2CA)x+G_1\hat{x}+(F_2CB-I)d\\ &\quad+G_2\dot{\hat{x}}+(F_2C+G_3)\Gamma(u,y)\end{aligned} \tag{6}$$

where $e_d=\hat{d}-d$. From Eq. (6), we can see that if there exists an F_2 that satisfies $F_2CB-I_{n_d\times n_d}=0$, then the effect of disturbance d can be eliminated. Let $G_2=0$, $G_1=-(F_1C+F_2CA)$ and $G_3=-F_2C$, Eq. (6) becomes

$$e_d=-(F_1C+F_2CA)e \tag{7}$$

Substituting Eq. (7) into Eq. (3), we have

$$\dot{e}=(A-B(F_1C+F_2CA)-KC)e \tag{8}$$

If $A-B(F_1C+F_2CA)-KC$ is Hurwitz, e will converge exponentially to zero. From Eq. (7), e_d will also converge exponentially to zero. □

Unfortunately, $(A-B(F_1C+F_2CA),C)$ may not always be an observable pair. If we ignore the nonlinear part of the plant (which will be canceled and therefore does not play a key role in the observer), it turns out that the open-loop zeros of the plant (A,B,C,0) are poles of the error dynamics. This fact is proven in the following Lemma.

Lemma 3.2: Open-loop zeros of the plant (A,B,C,0) are poles of the error dynamics $\dot{e}=(A-B(F_1C+F_2CA)-KC)e$.

Proof: The following four facts are required to prove this Lemma:

Fact 1: For two square matrices A and B, $\det(AB)=\det(BA)=\det A\cdot\det B$.

Fact 2: For any $n\times m$ matrix A and $m\times n$ matrix B, $\det(I_n-AB)=\det(I_m-BA)$

Fact 3: For any four matrices A, B, C and D of proper size, we have

$$\det\begin{pmatrix}A & D\\ C & B\end{pmatrix}=\det A\cdot\det(B-CA^{-1}D)$$

Fact 4: $F_2CB=I_{n_d\times n_d}$

Fact 5: If a matrix $\breve{A}$ is obtained from another matrix A by adding a multiple of any row (or column) to another, then $\det\breve{A}=\det A$.

The poles of the error dynamics satisfy $\det[pI-A+B(F_1C+F_2CA)+KC]=0$. From the five Facts shown above, we have (for all the poles of the error dynamics p)

$$\begin{aligned}&\det[pI-A+B(F_1C+F_2CA)+KC]\\ &\overset{(1)}{=}\det(pI-A+KC+BF_2CA)\\ &\quad\cdot\det[I+(pI-A+KC+BF_2CA)^{-1}BF_1C]\\ &\overset{(2)}{=}\det(pI-A+KC+BF_2CA)\\ &\quad\cdot\det[I+F_1C(pI-A+KC+BF_2CA)^{-1}B]\\ &\overset{(3)}{=}\det\begin{bmatrix}pI-A+KC+BF_2CA & -B\\ F_1C & I_{n_d\times n_d}\end{bmatrix}\\ &\overset{(4)}{=}\det\begin{bmatrix}pI-A+KC+BF_2CA & -B\\ F_1C & F_2CB\end{bmatrix}\\ &\overset{(5)}{=}\det\begin{bmatrix}pI-A+KC+BF_2CA & -B\\ F_2C(pI+KC)+F_1C & 0\end{bmatrix}=0\end{aligned} \tag{9}$$

where the number above each equality sign indicates the "Fact" used in derivation. For any open-loop zero z of the plant, we have

$$\det\begin{bmatrix}zI-A & -B\\ C & 0\end{bmatrix}=0.$$

Since

$$\begin{aligned}\det\begin{bmatrix}zI-A & -B\\ C & 0\end{bmatrix}&\overset{(5)}{=}\det\begin{bmatrix}zI-A+KC+BF_2CA & -B\\ C & 0\end{bmatrix}\\ &\overset{(5)}{=}\det\begin{bmatrix}zI-A+KC+BF_2CA & -B\\ (pF_2+F_2CK+F_1)C & 0\end{bmatrix}\end{aligned}$$

From Eq. (9), it is obvious all open-loop zeros (z) are poles of the error dynamics (p). □

The fact stated in Lemma 3.2 is the main reason we call the proposed estimation schemes "inverse-dynamics-based" observers. It should be noted that for SISO systems, the condition $F_2CB-I_{n_d\times n_d}=0$ can be satisfied if and only if $CB\neq 0$; i.e., when the system's relative degree is equal to one. This is exactly the same constraint obtained in [18,19]. This constraint implies that to use the design illustrated in **Lemma 3.1**, we need to, if possible, choose output measurements to satisfy $F_2CB - I_{n_d \times n_d} = 0$. Furthermore, since $\mathrm{rank}(F_2CB) \leqslant \min[\mathrm{rank}(F_2), \mathrm{rank}(C), \mathrm{rank}(B)]$ and $F_2 \in R^{n_d \times n_y}$, we have a necessary (but not sufficient) condition that $n_y \geqslant n_d$. In other words, the number of output signals must be larger than or equal to the number of disturbance signals. The same conclusion was also reached by Stein and Park. In comparison with existing methodologies, however, the observer proposed above has the following advantages

1. It does not require any special canonical form (thus, no coordinate transformation).
2. The rank condition and observer gains are clearly defined.
3. The convergence rate of states and disturbance estimations can be adjusted by tuning the design factors K and F_1 (governed by Eqs. (7) and (8)), up to the bandwidth limited by the zeros of the plant.

Many physical systems fail to satisfy the conditions for **Case 1** systems. Therefore, a design process for systems with more relaxed "invertibility" condition is proposed below.

Case 2: For any F_2, $F_2CB - I_{n_d \times n_d} \neq 0$

If we cannot find a finite F_2 that satisfies $F_2CB - I_{n_d \times n_d} = 0$, a different observer scheme must be developed. Note that the following method can also be applied to systems that satisfy $F_2CB - I_{n_d \times n_d} = 0$ ("case 1" type).

Lemma 3.3: For the state and input observers shown in Eqs. (2) and (4). If we choose $G_1 = -(F_1C + B^+A)$, $G_2 = -(F_2C - B^+)$ and $G_3 = -B^+$, where B^+ is a left inverse of B. The error dynamics are then $M_e\dot{e} = A_e e$, where $M_e = (I + B(F_2C - B^+))$ and $A_e = (A - KC - B(F_1C + B^+A))$.

Proof: Since B has full column rank, from Eq. (1) we have

$$d = B^+\dot{x} - B^+Ax - B^+\Gamma(u,y) \tag{10}$$

Subtract Eq. (10) from (4), we have

$$e_d = (F_1C + B^+A)x + G_1\hat{x} + (F_2C - B^+)\dot{x} + G_2\dot{\hat{x}} + (G_3 + B^+)\Gamma(u,y) \tag{11}$$

where $e_d = \hat{d} - d$. By following the same procedure illustrated in **Lemma 3.1**, we can choose

$$G_1 = -(F_1C + B^+A) \tag{12}$$

$$G_2 = -(F_2C - B^+) \tag{13}$$

$$G_3 = -B^+ \tag{14}$$

so that Eq. (11) becomes

$$e_d = -(F_1C + B^+A)e - (F_2C - B^+)\dot{e} \tag{15}$$

Substituting Eq. (15) into Eq. (3), we have

$$M_e\dot{e} = A_e e \tag{16}$$

where

$$M_e = (I + B(F_2C - B^+)) \tag{17}$$

$$A_e = (A - KC - B(F_1C + B^+A)) \tag{18}$$

□

When M_e is nonsingular, F_1, F_2, and K should be selected such that $M_e^{-1}A_e$ becomes Hurwitz. In general, however, M_e may be singular and M_e^{-1} does not exist. In this case, the singular value decomposition (SVD) technique [22] needs to be used. Let

$$M_e = U\Sigma V^T \tag{19}$$

$$\Sigma = \begin{bmatrix} \sigma_M & 0 \\ 0 & 0 \end{bmatrix} \tag{20}$$

where $U \in R^{n \times n}$ and $V \in R^{n \times n}$ are unitary matrices, and $\sigma_M \in R^{n_M \times n_M}$ ($n_m \leqslant n$) is a positive-definite diagonal matrix. Substitute Eq. (19) into Eq. (16)

$$U\Sigma V^T\dot{e} = A_e e \tag{21}$$

Since U and V are unitary matrices, Eq. (21) can be rewritten as

$$\Sigma V^T\dot{e} = U^T A_e V V^T e \tag{22}$$

Let $z \equiv V^T e$, Eq. (22) becomes

$$\Sigma\dot{z} = U^T A_e V z \tag{23}$$

Using the notations of Eq. (20), Eq. (23) can be rewritten as

$$\begin{bmatrix} \sigma_M & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{24}$$

where $z_1 \in R^{n_M}$, $z_2 \in R^{n-n_M}$,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \equiv U^T A_e V,$$

and $A_{11}, \ldots A_{22}$ are defined according to the matrix partition. From Eq. (24), we have

$$\dot{z}_1 = \sigma_M^{-1}A_{11}z_1 + \sigma_M^{-1}A_{12}z_2 \tag{25}$$

$$0 = A_{21}z_1 + A_{22}z_2 \tag{26}$$

Equations (25) and (26) give the error dynamics for the case when M_e is singular. The observer design is thus to choose F_1, F_2 and K such that the origin under Eqs. (25)–(26) is stable. The necessary and sufficient condition for exponential stability is shown in the following two facts.

Fact 3.4 Given a linear systems

$$\begin{bmatrix} \dot{z}_1 \\ \varepsilon\dot{z}_2 \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix},$$

suppose that B_{22} and $B_{11} - B_{12}B_{22}^{-1}B_{21}$ are both Hurwitz, then there exists a positive number ε_o such that $\forall \varepsilon \in (0, \varepsilon_o)$,

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21}/\varepsilon & B_{22}/\varepsilon \end{bmatrix}$$

is Hurwitz.

Proof: This is a well known singular-perturbation result, and the proof can be found in books such as Vidyasagar [23]. □

Fact 3.5 The error dynamics $M_e\dot{e} = A_e e$ are stable if and only if both A_{22} and $A_{11} - A_{12}A_{22}^{-1}A_{21}$ are Hurwitz, where

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \equiv U^T A_e V,$$

and $M_e = U\Sigma V^T$ is the SVD of M_e.

Proof: This can be shown easily by using Fact 3.5, and the fact that σ_M is positive definite.

□

From **Fact 3.5**, F_1, F_2 and K need to be chosen to make $A_{11} - A_{12}A_{22}^{-1}A_{21}$ and A_{22} Hurwitz, which then guarantees $z \to 0$ (i.e., $e \to 0$, since $z = V^T e$ and V is nonsingular). Sufficient conditions to guarantee A_{22} and $A_{11} - A_{12}A_{22}^{-1}A_{21}$ to be Hurwitz have not been identified, since these two matrices depend on several design factors and their singular value decomposition results. In the observer design for **Case 2** systems, we found that usually we have sufficient design degrees of freedom (i.e., F_1, F_2 and K). The nonuniqueness of B^+ (the left inverse of B) also provides extra design degree of freedom. This fact will be shown in a design example in the following section.

It should be noted that when the plant transfer function (from disturbance to output) has a relative degree higher than 1, the design procedure shown in Case 2 has to be used. The fact that $G_2 \neq 0$ in Case 2 implies that the rate of change of the estimated states are necessary to reconstruct the disturbance d (in an exponential fashion). When the plant relative degree is one, exponentially-converging estimation is achievable without using that piece of information.

Since $A_e = (A - KC - B(F_1C + B^+A))$ in Eq. (18) is very similar to the state matrix of Eq. (8), it is tempting to postulate that by following the same procedure shown in **Lemma 3.2**, it can be shown that the open-loop zeros of the plant are poles of A_e. Due to the nonuniqueness of the generalized left-inverse matrix B^+, it is difficult to prove that conjecture. When M_e is nonsingular, however, that "inverse dynamics" property is easy to prove.

Lemma 3.6: When M_e is nonsingular, open-loop zeros of the plant (A,B,C,0) are poles of the error dynamics $\dot{e} = M_e^{-1}A_e e$.

Proof: The poles of the error dynamics satisfy $\det[pI - M_e^{-1}A_e] = 0$.

The five facts illustrated in Lemma 3.2 will be used in the following without restating them. From Eqs. (17) and (18), we have

$$\det\{pI - [I + B(F_2C - B^+)]^{-1}[A - KC - B(F_1C + B^+A)]\} = 0$$

$$\Rightarrow \det\begin{bmatrix} I + B(F_2C - B^+) & A - KC - B(F_1C + B^+A) \\ I & pI \end{bmatrix} = 0$$

$$\Rightarrow \det\begin{bmatrix} I + B(F_2C - B^+) & -pI - pB(F_2C - B^+) + A - KC - B(F_1C + B^+A) \\ I & 0 \end{bmatrix} = 0$$

$$\Rightarrow \det[pI - A + KC + B(F_1C + B^+A) + pB(F_2C - B^+)] = 0$$

$$\Rightarrow \det[pI - A + KC + B(F_1C + B^+A)] \cdot \det\{I + [pI - A + KC + B(F_1C + B^+A)]^{-1}pB(F_2C - B^+)\} = 0$$

$$\Rightarrow \det\begin{bmatrix} pI - A + KC + B(F_1C + B^+A) & -pB \\ F_2C - B^+ & B^+B \end{bmatrix}$$

$$= \det\begin{bmatrix} pI - A + KC + B(F_1C + B^+A) & -pB \\ \left[F_2 + \frac{1}{p}(B^+K + F_1)\right]C & 0 \end{bmatrix} = 0$$

It is then straightforward to show that any open-loop zero that satisfies

$$\det\begin{bmatrix} zI - A & -B \\ C & 0 \end{bmatrix} = 0$$

also satisfies

$$\det\begin{bmatrix} zI - A + KC + B(F_1C + B^+A) & -pB \\ \left[F_2 + \frac{1}{p}(B^+K + F_1)\right]C & 0 \end{bmatrix} = 0.$$

In other words, open-loop zeros are poles of the error dynamics $\dot{e} = M_e^{-1}A_e e$. □

3.2 H_∞ Filtering Problem. To better assess the performance of the proposed observer, it needs to be compared against existing algorithms. In this paper, we choose to use the standard H_∞ filtering scheme [24,25] as the benchmark. In the H_∞ filtering setting, the plant dynamics are assumed to be described by

$$\dot{x}(t) = Ax(t) + Bd(t)$$

$$z(t) = C_1x(t) + D_{11}d(t) \tag{27}$$

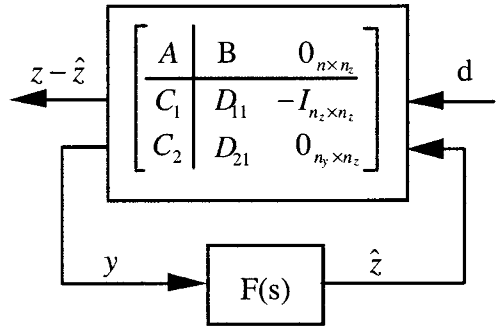

Fig. 1 Diagram of a standard H_∞ filtering problem

$$y(t) = C_2x(t) + D_{21}d(t)$$

where $z(t) \in R^{n_z}$ denotes the vector to be estimated. The problem is then to design a causal filter $F(s)$ such that the estimation of $z(t)$ (i.e., $\hat{z}(t)$) is obtained in the H_∞-(sub)optimal sense based on the measurement $y(t)$ [26,27]. This problem can be formulated in the LFT framework, and its diagram is shown in Fig. 1. The filter $F(s)$ can be found easily by using the H_∞-solver available in the MATLAB robust control toolbox, such as the $hinf(\,)$ command.

4 Design Examples

4.1 An SISO System. The hypothetical two-state system studied in this example is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} d \quad y = x_2 \tag{28}$$

where d is a scalar disturbance. Since $C = [0\ 1]$ and $B = [1\ 0]^T$, $CB = 0$. Therefore, only the Case 2 algorithm (shown in **Lemma 3.3**) is applicable. Due to the fact that B is a column vector, the left inverse B^+ is not unique. All row vectors in the form $[1\ \rho]$, $\rho \in R$ are candidates for B^+. If we keep B^+ in this general form, the parameter ρ becomes an extra design degree of freedom. From Eqs. (12)–(14), we have $G_1 = -[a_1 + \rho a_3\ \ F_1 + a_2 + \rho a_4]$, $G_2 = [1\ \rho - F_2]$ and $G_3 = [-1\ -\rho]$. The error dynamics for the state observer are $M_e\dot{e} = A_e e$, where

$$M_e = \begin{bmatrix} 0 & F_2 - \rho \\ 0 & 1 \end{bmatrix}, \quad A_e = \begin{bmatrix} -\rho a_3 & -k_1 - \rho a_4 - F_1 \\ a_3 & a_4 - k_2 \end{bmatrix}$$

and k_1 and k_2 are the elements of the observer gain $K(=[k_1\ k_2]^T)$. Next, we need to find the SVD for $M_e\ (= U\Sigma V^T)$. It is well known that the unitary matrices U and V are the orthonormal matrix of the eigenvectors of $M_eM_e^T$ and $M_e^TM_e$, respectively. For this plant, we have

$$V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} \sqrt{(F_2 - \rho)^2 + 1} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{and}\quad U = \frac{1}{\sqrt{(F_2 - \rho)^2 + 1}}\begin{bmatrix} F_2 - \rho & 1 \\ 1 & -(F_2 - \rho) \end{bmatrix}.$$

Therefore,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = U^TA_eV = \frac{1}{\sqrt{(F_2 - \rho)^2 + 1}}\begin{bmatrix} -(F_2 - \rho)(k_1 + \rho a_4 + F_1) + a_4 - k_2 & a_3 - \rho a_3(F_2 - \rho) \\ -k_1 - F_1 - F_2a_4 + F_2k_2 - \rho k_2 & -a_3F_2 \end{bmatrix}$$

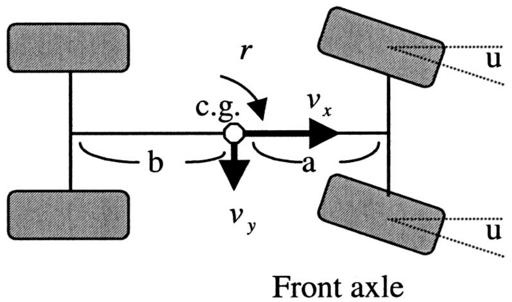

Fig. 2 The bicycle vehicle handling model

The two conditions for Fact 3.6 (A_{22} and $A_{11}-A_{12}A_{22}^{-1}A_{21}$ are Hurwitz) then become

$$a_3F_2>0 \tag{29}$$

and

$$\rho^3(a_3k_2)+\rho^2(a_3k_1+a_3F_1-2a_3F_2k_2)+\rho(F_2^2a_3k_2-2a_3F_2k_1-2a_3F_1F_2+a_3k_2)+(a_3k_1+a_3F_1+a_3F_2^2k_1+a_3F_1F_2^2)>0 \tag{30}$$

The following analysis are obtained based on Eqs. (29) and (30).

1. Equations (29) and (30) can be satisfied only if $a_3\neq 0$, which implies that the system has to be observable. When $a_3=0$, the measured state x_2 is not affected by x_1 and the observability matrix becomes singular. Hence, no exponentially decaying observer can be designed. Fortunately, this unfavorable condition has been excluded by **Assumption 2.1**. In other words, we have $a_3\neq 0$ due to the observability assumption.
2. From Eq. (29), F_2 cannot be zero. Since F_2 is the observer gain for $\dot{y}$, output derivative feedback is necessary in order to construct exponentially converging observer.
3. For the special case $\rho=0$, Eqs. (29) and (30) reduce to $a_3F_2>0$ and $(k_1+F_1)/F_2>0$, which are easy to satisfy.
4. When $k_2\neq 0$, Eq. (30) can always be satisfied, no matter what values we have selected for the other observer gains. For example, for any observer design, there exists $M\gg 1$, such that $\rho=sgn(a_3k_2)\cdot M$ will satisfy Eq. (30). In other words, the requirement on the other design parameters (F_1, F_2 and K) is even more relaxed when $\rho\neq 0$.

4.2 Vehicle Lateral Speed and External Disturbance Estimation. Vehicle lateral speed is an important piece of information for the design of ground vehicle active safety systems such as vehicle stability control systems [28], lane departure warning systems [29], etc. Vehicle lateral speed can be measured by using vision-based or radar-based sensors, but these sensors are currently too costly (~$5,000–$20,000) for production vehicles. Therefore, observer techniques to estimate vehicle lateral speed from yaw rate measurement (~$50) are an attractive alternative. A widely acknowledged problem is that road superelevation and wind gust disturbances may cause significant errors in the estimation. In this design example, we will apply the observer scheme presented in Section 3 to estimate vehicle lateral speed in the presence of external disturbances.

The well-known bicycle model [30] is used to describe vehicle lateral and yaw dynamics. From Fig. 2, assuming linear tire model, we have

$$\begin{bmatrix}\dot{v}_y\\ \dot{r}\end{bmatrix}=\begin{bmatrix}-\frac{1}{m}\left(\frac{2C_f+2C_r}{v_x}\right) & \frac{1}{m}\left(-mv_x+\frac{-2aC_f+2bC_r}{v_x}\right)\\ \frac{1}{I_z}\left(\frac{-2aC_f+2bC_r}{v_x}\right) & \frac{1}{I_z}\left(\frac{-2a^2C_f+2b^2C_r}{v_x}\right)\end{bmatrix}\times\begin{bmatrix}v_y\\ r\end{bmatrix}+\begin{bmatrix}\frac{2C_f}{m}\\ \frac{2aC_f}{I_z}\end{bmatrix}u+\begin{bmatrix}1\\0\end{bmatrix}d\equiv Ax+B_uu+Bd$$

$$y=r\equiv Cx \tag{31}$$

where v_y is the vehicle lateral speed, r is the measured yaw rate, v_x is vehicle longitudinal speed, and u is the vehicle front wheel steering angle—the "control" input possibly from a human driver or automatic steering systems. The vehicle mass and yaw moment of inertia are denoted as m and I_z, respectively. The vehicle center of gravity is assumed to locate at a distance a and b from the front and rear axles. The front and rear tire cornering stiffness are denoted as C_f and C_r, respectively. The resultant external disturbance is denoted as d which includes the effects of wind gust and the gravity force induced by road superelevation. It is assumed that d acts on the vehicle's center of gravity. Since $CB=0$, we will have to use the methods described in **Lemma 3.3**.

Equation (31) is essentially the same as Eq. (28), except that it has control (steering) input, and that we have associated the plant with a real plant (the vehicle), instead of imaginary symbols. Therefore, the observer design process follows the procedure shown in section 4.1 directly. By assuming the following nominal vehicle parameters: $C_f=51{,}000$ N/rad, $C_r=47{,}000$ N/rad, $m=1700$ kg, $a=1.14$ m, $b=1.41$ m and $I_z=3200$ kg m^2, and vehicle speed $v_x=15$ m/s, we have (by using the notations shown in Eq. (28)) $a_1=-9.754$, $a_2=-14.03$, $a_3=0.5901$ and $a_4=-12.622$. The free parameter ρ in B^+ is chosen to be zero since we do not need that extra design degree of freedom. The inequality constraints Eq. (29) and (30) then reduce to two simple constraints: $F_2>0$ and $k_1+F_1>0$.

Although the conditions for exponentially converging observer are easy to achieve, specifications from transient performance requirement can be formed to dictate the observer gain selection. For this example, the error dynamics are governed by: $\dot{e}_2=[-(k_1+F_1)/F_2]/e_2$ and $e_1=[k_1+F_1+F_2(a_4-k_2)]/(F_2a_3)e_2$. The first equation is guaranteed to be stable (since $F_2>0$ and $k_1+F_1>0$). The second equation described the relative size of the unmeasured state (lateral speed) estimation error with respect to that of the measured state (yaw rate). Apparently, if vehicle lateral speed estimation is of concern, the free parameters should be selected such that the magnitude of $[k_1+F_1+F_2(a_4-k_2)]/F_2$ becomes as small as possible. This can be formulated as a nonlinear programming optimization problem. The following inequality constraints were imposed: (i) $F_2>0$ and $k_1+F_1>0$, (ii) observer poles should have real parts less than -12, (iii) $k_1+F_1/F_2>3$, and (iv) magnitudes of all the free observer gains are bounded ($|F_1|\leqslant 10$, $|k_2|\leqslant 50$, and $|F_2|\leqslant 1$). The best design (which minimizes the absolute value of $[k_1+F_1+F_2(a_4-k_2)]/F_2$ under these constraints was found to be $K=[k_1\ k_2]^T=[47\ 10]^T$, $F_1=-10$ and $F_2=1$. These observer gains are used for the simulations described below.

The scenario used to verify this observer design in simulations is as follows: the vehicle is assumed to be driving at a constant speed of 15 m/s in a long constant radius curve. A constant steering (u) of 0.33 degree is applied. The external disturbance includes the effect of both the road superelevation and the wind gust disturbances, and is assumed to be a combination of a trapezoidal function (superelevation=5 degree between t=1~4 second) and three sinusoidal signals (wind gust force at three frequencies—3, 10, and 35 rad/s). The yaw rate and steering angle are both assumed to be measured at a rate of 100 Hz, based on which the

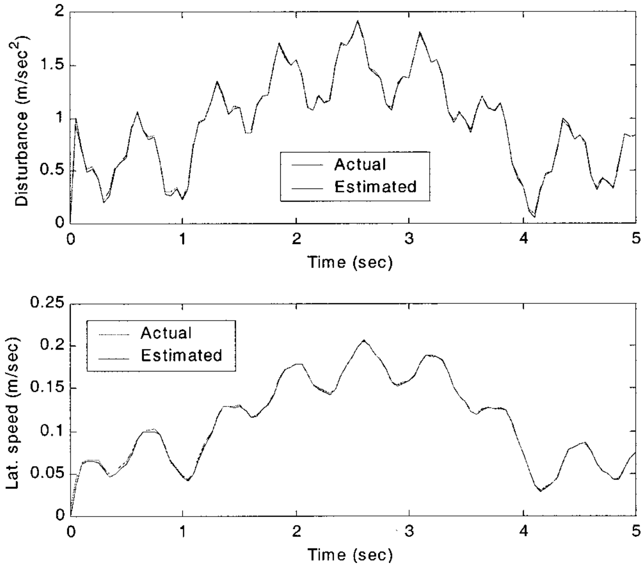

Fig. 3 Disturbance and lateral speed estimation results (inverse-dynamics approach)

continuous-time algorithm is implemented. The derivative of output signal is obtained by using simple backward difference based on the 100 Hz measurement. The disturbance and state estimation results under ideal condition (no uncertainties) are shown in Fig. 3. The simulation results show that the estimations are quite accurate. This is perhaps not so surprising since no uncertainties have been introduced. The estimation error thus decays quickly to zero as predicted in the theory.

If we apply the H_∞ filter algorithm to estimate vehicle lateral speed and disturbance directly, the results (Fig. 4) are similar but slightly worse compared to the results of Fig. 3. It can be seen that the estimate of lateral speed is quite good, the disturbance estimation is more oscillatory than that of Fig. 3.

The robustness of the proposed algorithm is subsequently studied by investigating the effect of measurement noise and model mismatch. The uncertainties include (1) yaw rate measurement is assumed to be contaminated by white noise (0.001 rad/s standard deviation) plus a sinusoidal noise (0.001 rad/s amplitude, 1 Hz); (2) Due to steering system misalignment, an offset of 0.01 degree is assumed to exist; and (3) The true vehicle mass and yaw moment of inertia are assumed to be 20% higher than nominal values, and the normalized tire cornering stiffness is assumed to be 30% below the nominal value. These uncertainties represent a severely perturbed operating condition. A standard (discrete-time) fifth-order low-pass filter was added to the filter out the white noise in yaw rate measurement. The estimation deteriorates noticeably (Fig. 5) but the performance is still quite reasonable.

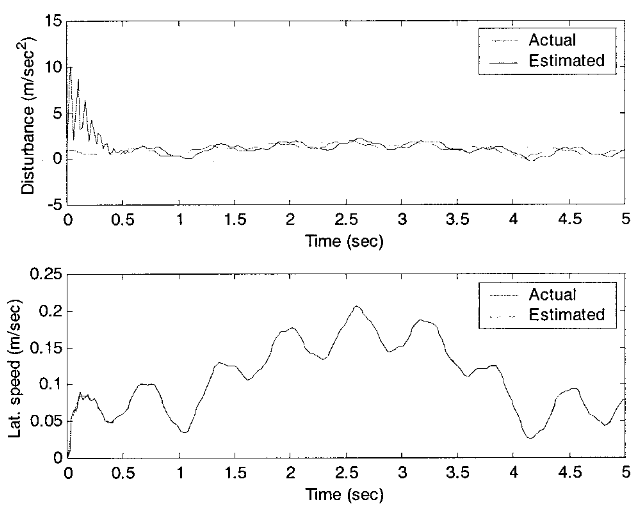

Fig. 4 Disturbance and lateral speed estimation results (H_∞ approach)

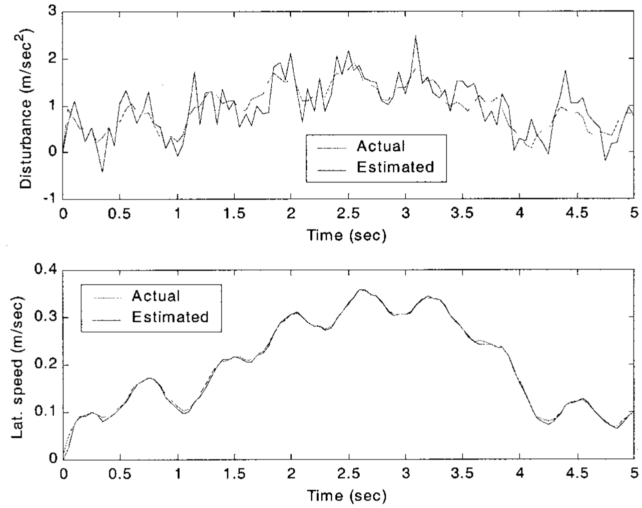

Fig. 5 Disturbance and lateral speed estimation results (inverse-dynamics approach, combined uncertainties)

5 Conclusions

In this paper, two output feedback observer schemes were proposed to estimate unknown states and disturbances simultaneously for a special class of nonlinear time invariant systems. The algorithms were developed based on the inverse dynamic concept. Depending on the relationship between system output and disturbance input matrices, two distinct algorithms were proposed. Guidelines for the design of the disturbance observer gain matrices were presented. Sufficient conditions for the asymptotic convergence of estimation errors were also given. The proposed algorithms require neither disturbance models, nor state coordinate transformations, and the observer can be applied to SISO systems with relative degree higher than 1. Two design examples were presented to illustrate the design procedure. In particular, the proposed observers were successfully applied to a vehicle lateral speed and disturbance estimation problem. Simulation results show that even under an array of severe system uncertainties, the vehicle lateral speed and external disturbances are estimated accurately.

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