Lawrence Mianzo
Graduate Student,
Visteon Asia Pacific,
Electronics Development Center,
Hiroshima, Japan
e-mail: lmianzo@ford.com

Huei Peng
Assistant Professor,
Department of Mechanical Engineering
and Applied Mechanics,
Ann Arbor, MI 48109-2125
e-mail: hpeng@umich.edu

A Unified Hamiltonian Approach for LQ and H_∞ Preview Control Algorithms

A framework for solving both the continuous and discrete-time LQ and H_∞ preview control algorithms is presented in this paper. The tracking control of an automotive durability test rig is used as an application example. Simulation results are presented to illustrate the effectiveness of the preview control algorithms.

1 Introduction

Much work has focused on the H_∞ feedback control problem since the landmark 'DGKF' state-space solution was presented by Doyle et al. (1989). Most additional work has focused on the use of single-degree-of-freedom controller design, where both stability and performance requirements are satisfied by a single controller block. In Yaesh and Shaked (1991), as well as several earlier work (e.g., Soroka and Shaked, 1986; Grimble, 1988; Hara and Sugie, 1988), two-degree of freedom (2DOF) H_∞ controllers were proposed. In their formulation, two control blocks are designed separately for better trade-off between stability and performance. The 2DOF formulations often increase the order of the controller as they cannot, in general, be solved in the standard H_∞ framework. For example, in the spectral factorization approach proposed by Yaesh and Shaked (1991), the feedback block is first designed for robust stability and noise requirement, the feedforward block is then designed to improve tracking performance. The order of the feedforward controller, however, equals to the sum of the order of feedback controller and the augmented plant.

Preview control algorithms have been widely used to improve control performance when future desired output or exogenous disturbance is available. LQ-based preview control theory was first developed in the 1970's. In an early derivation (Tomizuka and Whitney, 1975) the optimal preview control signal was found to consist of a feedback and two preview terms. LQ-based preview control algorithms have been applied to a wide range of applications with significant success (e.g., Peng and Tomizuka, 1993).

H_∞-Preview control algorithms have gained increased interest recently. In Moran et al. (1996), a modified game Riccati Equation is used and the feedforward control is assumed unchanged from the LQ-preview control formulation. In Ma and Peng (1996), a two-player game theory approach is used. The signals of one of the players is assumed to be previewable by the other player. In Mianzo and Peng (1997), a Hamiltonian based method was developed to allow for the simultaneous design of feedback and preview control laws. In this paper, this approach is generalized to a framework suitable for continuous and discrete-time, LQ and H_∞, and tracking and regulation problems.

The remainder of this paper is organized as follows: in Section 2, the continuous-time problem is formulated. The solution is then given in Section 3. The discrete-time results are shown in Sections 4 and 5. Simulation results for the tracking control of an automotive durability test rig are shown in Section 6. Finally, conclusions are drawn in Section 7.

Contributed by the Dynamic Systems and Control Division for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received by the Dynamic Systems and Control Division March 25, 1998. Associate Technical Editor: J. E. Colgate.

2 Continuous-Time Hamiltonian Formulation

The continuous linear time-invariant system studied in this paper is assumed to be described by the following realization:

$$\dot{x} = Ax + B_1 w + B_2 u + B_p w_p$$

$$z = C_1 x + D_{11} w + D_{12} u + D_{1p} w_p \quad (1)$$

where x is the state vector, u is the control vector, z is the performance vector, w is the non-previewable, and w_p is the previewable disturbance. w_p can be either the desired trajectory or previewable external disturbance. w includes model uncertainty, measurement noise and other unknown disturbances. The cost function to be minimized by the control vector is assumed to be $J = \frac{1}{2}\int_0^{t_f} [z^T z + u^T u - \gamma^2 w^T w] dt$ and the Hamiltonian function is defined as

$$H(x, \lambda, t) = \tfrac{1}{2}[z^T z + u^T u - \gamma^2 w^T w] + \lambda^T (Ax + B_1 w + B_2 u + B_p w_p) \quad (2)$$

Therefore, optimality $[(\partial H/\partial u) = 0$ and $(\partial H/\partial w) = 0]$ is achieved when

$$u = \Pi_1 (D_{12}^T C_1 + D_{12}^T D_{11} \Delta_2 D_{11}^T C_1) x + \Pi_1 (D_{12}^T D_{11} \Delta_2 D_{11}^T D_{1p} + D_{12}^T D_{1p}) w_p + \Pi_1 (D_{12}^T D_{11} \Delta_2 B_1^T + B_2^T) \lambda \equiv g_1 x + g_2 w_p + g_3 \lambda \quad (3)$$

$$w = \Pi_2 (D_{11}^T C_1 + D_{11}^T D_{12} \Delta_1 D_{12}^T C_1) x + \Pi_2 (D_{11}^T D_{12} \Delta_1 D_{12}^T D_{1p} + D_{11}^T D_{1p}) w_p + \Pi_2 (D_{11}^T D_{12} \Delta_1 B_2^T + B_1^T) \lambda \equiv g_4 x + g_5 w_p + g_6 \lambda \quad (4)$$

where $\Delta_1 = -(I + D_{12}^T D_{12})^{-1}$, $\Pi_1 = -(I + D_{12}^T D_{11} \Delta_2 D_{11}^T D_{12} + D_{12}^T D_{12})^{-1}$, $\Delta_2 = (\gamma^2 - D_{11}^T D_{11})^{-1}$ and $\Pi_2 = (\gamma^2 - D_{11}^T D_{12} \Delta_1 D_{12}^T D_{11} - D_{11}^T D_{11})^{-1}$. Substitute Eqs. (3) and (4) into (1), we have

$$\begin{aligned}\dot{x} = Ax &+ B_1 \Pi_2 (D_{11}^T C_1 + D_{11}^T D_{12} \Delta_1 D_{12}^T C_1) x \\ &+ B_2 \Pi_1 (D_{12}^T C_1 + D_{12}^T D_{11} \Delta_2 D_{11}^T C_1) x \\ &+ B_1 \Pi_2 (D_{11}^T D_{12} \Delta_1 D_{12}^T D_{1p} + D_{11}^T D_{1p}) w_p \\ &+ B_2 \Pi_1 (D_{12}^T D_{11} \Delta_2 D_{11}^T D_{1p} + D_{12}^T D_{1p}) w_p \\ &+ B_1 \Pi_2 (D_{11}^T D_{12} \Delta_1 B_2^T + B_1^T) \lambda \\ &+ B_2 \Pi_1 (D_{12}^T D_{11} \Delta_2 B_1^T + B_2^T) \lambda \quad (5)\end{aligned}$$

Finally, using the fact $-\dot{\lambda}^T = \partial H/\partial x$, we have

$$\dot{\lambda} = [\gamma^2 g_4^T g_4 - g_1^T g_1 - \Pi_3 C_1 - \Pi_3 D_{11} g_4 - \Pi_3 D_{12} g_1]x + [\gamma^2 g_4^T g_6 - A^T - \Pi_3 D_{11} g_6 - \Pi_3 D_{12} g_3 - g_1^T g_3 - g_1^T B_2^T - g_4^T B_1^T]\lambda + [\gamma^2 g_4^T g_5 - \Pi_3 D_{11} g_5 - \Pi_3 D_{12} g_2 - \Pi_3 D_{1p} - g_1^T g_2] w_p \quad (6)$$

where $\Pi_3 = C_1^T + (C_1^T D_{11} + C_1^T D_{12}\Delta_1^T D_{12}^T D_{11})\Pi_2^T D_{11}^T + (C_1^T D_{12} + C_1^T D_{11}\Delta_2^T D_{11}^T D_{12})\Pi_1^T D_{12}^T$. The state and costate equation can then be written in a standard Hamiltonian matrix form:

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ \chi & \delta \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} + \begin{bmatrix} M \\ N \end{bmatrix} w_p \quad (7)$$

where

$$\alpha = A + B_1 \Pi_2 (D_{11}^T C_1 + D_{11}^T D_{12} \Delta_1 D_{12}^T C_1) + B_2 \Pi_1 (D_{12}^T C_1 + D_{12}^T D_{11} \Delta_2 D_{11}^T C_1)$$

$$\beta = -B_1 \Pi_2 (D_{11}^T D_{12} \Delta_1 B_2^T + B_1^T) - B_2 \Pi_1 (D_{12}^T D_{11} \Delta_2 B_1^T + B_2^T)$$

$$\chi = \gamma^2 g_4^T g_4 - g_1^T g_1 - \Pi_3 C_1 - \Pi_3 D_{11} g_4 - \Pi_3 D_{12} g_1$$

$$\delta = \gamma^2 g_4^T g_6 - A^T - \Pi_3 D_{11} g_6 - \Pi_3 D_{12} g_3 - g_1^T g_3 - g_1^T B_2^T - g_4^T B_1^T$$

$$M = B_1 \Pi_2 (D_{11}^T D_{12} \Delta_1 D_{12}^T D_{1p} + D_{11}^T D_{1p}) + B_2 \Pi_1 (D_{12}^T D_{11} \Delta_2 D_{11}^T D_{1p} + D_{12}^T D_{1p}) + B_p$$

$$N = \gamma^2 g_4^T g_5 - \Pi_3 D_{11} g_5 - \Pi_3 D_{12} g_2 - \Pi_3 D_{1p} - g_1^T g_2 \quad (8)$$

After tedious but straightforward derivation, it was found that $\delta = -\alpha^T$.

3 LQ and H_∞ Continuous-Time Preview Control Algorithm

Equation (7) shows a two point boundary value problem with mixed boundary conditions, a difficult problem to solve. A common approach is to assume a solution, and then find the constraints imposed by the optimal condition for this particular form. When w_p is previewable within a preview window, i.e., at time t, the signal $w_p(\tau)$, $\tau \in [t, t + t_{la}]$ is known, one possible solution form inspired by previous LQ results (Tomizuka and Whitney, 1975) is

$$\lambda = Px + \int_0^{t_{la}} F_1(t, \tau) w_p(t + \tau) d\tau + F_2(t) w_p(t + t_{la}) \quad (9)$$

Differentiating both sides of Eq. (9) with respect to time, we obtain

$$\frac{dP}{dt} x + P\left[\alpha x - \beta\left(Px + \int_0^{t_{la}} F_1(t, \tau) w_p(t + \tau) d\tau + F_2(t) w_p(t + t_{la})\right) + M w_p(t)\right] + \int_0^{t_{la}} F_1(t, \tau) \frac{d}{dt} w_p(t + \tau) d\tau + \int_0^{t_{la}} \frac{d}{dt} F_1(t, \tau) w_p(t + \tau) d\tau + \dot{F}_2(t) w_p(t + t_{la})$$

$$+ F_2(t) w_p(t + t_{la}) = \chi x + \delta\left(Px + \int_0^{t_{la}} F_1(t, \tau) w_p \times (t + \tau) d\tau + F_2(t) w_p(t + t_{la})\right) + N w_p(t) \quad (10)$$

An extra assumption about the disturbance signal is made to simplify Eq. (10):

$$\frac{dw_p}{d\tau} = A_w w_p(\tau) \quad \tau \geq t + t_{la} \quad (11)$$

Using Eq. (11), Eq. (10) can be rewritten as

$$\frac{dP}{dt} x + P\alpha x - P\beta P x - P\beta \int_0^{t_{la}} F_1(t, \tau) w_p(t + \tau) d\tau - P\beta F_2(t) w_p(t + t_{la})) + PM w_p(t) + F_1(t, \tau) w_p(t + \tau)|_0^{t_{la}} - \int_0^{t_{la}} \frac{d}{d\tau} F_1(t, \tau) w_p(t + \tau) d\tau + \int_0^{t_{la}} \frac{d}{dt} F_1(t, \tau) w_p(t + \tau) d\tau + \dot{F}_2(t) w_p(t + t_{la}) + F_2(t) A_w w_p(t + t_{la}) = \chi x + \delta P x + \delta \int_0^{t_{la}} F_1(t, \tau) w_p(t + \tau) d\tau + \delta F_2(t) w_p(t + t_{la}) + N w_p(t) \quad (12)$$

From the Principle of Optimality, by grouping similar terms in Eq. (12), we have

$$\frac{dP}{dt} + P\alpha - P\beta P - \chi - \delta P = 0$$

$$P(t_f) = 0 \quad (13)$$

$$-P\beta F_1(t, \tau) - \frac{\partial}{\partial \tau} F_1(t, \tau) + \frac{\partial}{\partial t} F_1(t, \tau) - \delta F_1(t, \tau) = 0$$

$$F_1(t, 0) = PM - N \quad (14)$$

$$-P\beta F_2(t) + F_1(t, t_{la}) + \dot{F}_2(t) + F_2(t) A_w - \delta F_2(t) = 0$$

$$F_2(t_f) = 0 \quad (15)$$

Fact 3.1 When $z = C_1 x$ and the preview time is zero, Eqs. (13)–(15) solve a standard H_∞ feedback control problems.

Proof: It has been shown (Basar and Bernhard, 1995) that the two-player game problem described in Eqs. (1)–(4) is equivalent to an H_∞ problem. When $z = C_1 x$, $D_{11} = D_{12} = D_{1p} = 0$. From Eq. (8), $\alpha = A$, $\beta = B_2 B_2^T - \gamma^{-2} B_1 B_1^T$, $\chi = -C_1^T C_1$, $\delta = -A^T$, $M = B_p$ and $N = -C_1^T D_{1p}$. Therefore, Eq. (13) becomes $\dot{P} + PA - P(B_2 B_2^T - \gamma^{-2} B_1 B_1^T) P + C_1^T C_1 + A^T P = 0$, which is the standard game Riccati Equation, i.e., it solves a corresponding H_∞ feedback control problem.

Fact 3.2 When $\gamma \to \infty$ and $z = C_1 x$, Eqs. (13)–(15) solve a LQ-preview regulation problem.

Proof: When $\gamma \to \infty$, $\Delta_2 = \Pi_2 = 0$. By assuming $z = C_1 x$, we have $D_{11} = D_{12} = D_{1p} = 0$, $\Delta_1 = \Pi_1 = -1$, $g_1 = g_2 = g_4 =$

$g_5 = g_6 = 0$, $g_3 = -B_2^T$ and $\Pi_3 = C_1^T$. Therefore, Eqs. (13)–(15) reduce to

$$\dot{P} + PA - PB_2B_2^TP + C_1^TC_1 + A^TP = 0$$

$$P(t_f) = 0$$

$$-PB_2B_2^TF_1(t,\tau) - \frac{\partial}{\partial\tau}F_1(t,\tau) + \frac{\partial}{\partial t}F_1(t,\tau) + A^TF_1(t,\tau) = 0$$

$$F_1(t,0) = PM - N$$

$$-PB_2B_2^TF_2(t) + F_1(t,t_{la}) + \dot{F}_2(t) + F_2(t)A_w + A^TF_2(t) = 0$$

$$F_2(t_f) = 0$$

which are identical to the equations for LQ-preview regulation control algorithms (Peng and Tomizuka, 1993).

The optimal preview control law consists of a feedback and two preview terms. (i.e., $u(t) = (g_1 + g_3P)x(t) + g_2w_p(t) + g_3[\int_0^{t_{la}} F_1(\tau)w_p(t+\tau)d\tau + F_2w_p(t+t_{la})]$). From Facts 3.1 and 3.2, the feedback term is identical to either LQ or H_∞ feedback control. Therefore, the stability and existence condition of a solution for the proposed preview control law are exactly the same as standard LQ or H_∞ algorithms (Anderson and Moore, 1990; Zhou and Doyle, 1998). The readers are referred to those sources and the details are omitted here. We now summarize the results of Sections 2 and 3 in the following theorem.

Theorem 3.3 For a continuous linear time-invariant plant shown in Eq. (1), if (i) (A, B_1) is controllable and (C_1, A) is observable; (ii) (A, B_2) is stabilizable; and (iii) $H \equiv \left[\begin{smallmatrix}\alpha & -\beta \\ \chi & -\alpha^T\end{smallmatrix}\right] \in dom(Ric)$, then an admissible H_∞ preview control algorithm is $u(t) = (g_1 + g_3P)x(t) + g_2w_p(t) + g_3[\int_0^{t_{la}} F_1(\tau)w_p(t+\tau)d\tau + F_2w_p(t+t_{la})]$. The control-gain matrices P, F_1, and F_2 are governed by Eqs. (13)–(15), and the matrices α, β, χ, δ, M and N are defined in Eq. (8), and g_1, g_2 and g_3 are defined in Eq. (3).

The three assumptions stated in Theorem 3.3 are necessary to ensure the existence of a stabilizing control, internal stability, and the existence of a positive solution of the Riccati Equation. Interested readers are referred to Doyle et al. (1989) for details.

It is a common practice to implement the steady-state solutions of Eqs. (13)–(15), which need to be solved numerically. When w_p is a scalar, the solutions can be obtained as follows: The feedback gain matrix P can be obtained from Algebraic Riccati Equation solvers such as the MATLAB $are(\,)$ command ($P = are(\alpha, -\beta, -\chi)$), where the fact $\delta = -\alpha^T$ is used. The preview gains are then $F_1(\tau) = e^{-(P\beta+\delta)\tau}(PM - N)$ and $F_2 = (P\beta + \delta - A_wI)^{-1}e^{-(P\beta+\delta)t_{la}}(PM - N)$, respectively.

4 Discrete-Time Hamiltonian Formulation

A general discrete linear time-invariant system is shown below

$$x(k+1) = Ax(k) + B_1w(k) + B_2u(k) + B_pw_p(k)$$

$$z(k) = C_1x(k) + D_{11}w(k) + D_{12}u(k) + D_{1p}w_p(k) \quad (16)$$

where the variables x, u, w, w_p and z are as defined in Section 2. Although the equations are a lot cumbersome compared with their continuous-time counterparts, the solution process is very similar. Therefore, only important equations are listed in the following. The cost function to be minimized is chosen to be $J = \frac{1}{2}\sum_0^{N_f}[z^Tz + u^Tu - \gamma^2w^Tw]$. The Hamiltonian function is thus

$$H(x,\lambda,k) = \tfrac{1}{2}[z(k)^Tz(k) + u(k)^Tu(k) - \gamma^2w^T(k)w(k)]$$
$$+ \lambda(k+1)^T(Ax(k) + B_1w(k) + B_2u(k) + B_pw_p(k)) \quad (17)$$

The optimal algorithms are then

$$u(k) = \Pi_1(D_{12}^TC_1 + D_{12}^TD_{11}\Delta_2D_{11}^TC_1)x(k)$$
$$+ \Pi_1(D_{12}^TD_{11}\Delta_2D_{11}^TD_{1p} + D_{12}^TD_{1p})w_p(k) + \Pi_1(D_{12}^TD_{11}\Delta_2B_1^T$$
$$+ B_2^T)\lambda(k+1) \equiv g_1x(k) + g_2w_p(k) + g_3\lambda(k+1) \quad (18)$$

$$w(k) = \Pi_2(D_{11}^TC_1 + D_{11}^TD_{12}\Delta_1D_{12}^TC_1)x(k)$$
$$+ \Pi_2(D_{11}^TD_{12}\Delta_1D_{12}^TD_{1p} + D_{11}^TD_{1p})w_p(k)$$
$$+ \Pi_2(B_1^T + D_{11}^TD_{12}\Delta_1B_2^T)\lambda(k+1)$$
$$\equiv g_4x(k) + g_5w_p(k) + g_6\lambda(k+1) \quad (19)$$

where $\Delta_1 = -(I + D_{12}^TD_{12})^{-1}$, $\Pi_1 = -(I + D_{12}^TD_{11}\Delta_2D_{11}^TD_{12} + D_{12}^TD_{12})^{-1}$, $\Delta_2 = (\gamma^2 - D_{11}^TD_{11})^{-1}$ and $\Pi_2 = (\gamma^2 - D_{11}^TD_{12}\Delta_1D_{12}^TD_{11} - D_{11}^TD_{11})^{-1}$. From the fact $\lambda^T(k) = (\partial H/\partial x(k))$, we have

$$\lambda(k) = [g_1^Tg_1 + \Pi_3C_1 + \Pi_3D_{11}g_4 + \Pi_3D_{12}g_1 - \gamma^2g_4^Tg_4]x(k)$$
$$+ [A^T + \Pi_3D_{11}g_6 + \Pi_3D_{12}g_3 + g_1^Tg_3$$
$$+ g_1^TB_2^T + g_4^TB_1^T - \gamma^2g_4^Tg_6]\lambda(k+1) + [\Pi_3D_{11}g_5$$
$$+ \Pi_3D_{12}g_2 + \Pi_3D_{1p} + g_1^Tg_2 - \gamma^2g_4^Tg_5]w_p(k) \quad (20)$$

where $\Pi_3 = C_1^T + (C_1^TD_{11} + C_1^TD_{12}\Delta_1^TD_{12}^TD_{11})\Pi_2^TD_{11}^T + (C_1^TD_{12} + C_1^TD_{11}\Delta_2^TD_{11}^TD_{12})\Pi_1^TD_{12}^T$. The combined system dynamics can then be written as

$$\begin{bmatrix} x(k+1) \\ \lambda(k) \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ \chi & \delta \end{bmatrix}\begin{bmatrix} x(k) \\ \lambda(k+1) \end{bmatrix} + \begin{bmatrix} M \\ N \end{bmatrix}w_p(k) \quad (21)$$

The matrices α, β, χ, δ, M and N are almost identical to their continuous-time counterparts defined in Eq. (8), except that χ, δ and N are defined with an extra minus sign (i.e., $\chi_{\text{discrete}} = -\chi_{\text{continuous}}$, etc.) Similar to the continuous-time case, it can be demonstrated that $\delta = \alpha^T$.

5 LQ and H_∞ Discrete-Time Preview Control Algorithm

The costate dynamics solution is assumed to be

$$\lambda(k) = P(k)x(k) + \sum_{n=0}^{N_{la}-1} F_1(k,n)w_p(k+n)$$
$$+ F_2(k)w_p(k+N_{la}) \quad (22)$$

where N_{la} is the number of preview steps. Similar to the continuous-time case, the disturbance outside of the preview are assumed to be described by $p(k+j+1) = A_wp(k+j)$, $j \ge N_{la}$. The gain matrices of Eq. (22) were then found to be governed by

$$P(k) = \chi + \delta[I + P(k+1)\beta]^{-1}P(k+1)\alpha$$
$$P(N) = 0 \quad (23)$$
$$F_1(k,n) = \delta[I + P(k+1)\beta]^{-1}F_1(k+1, n-1)$$

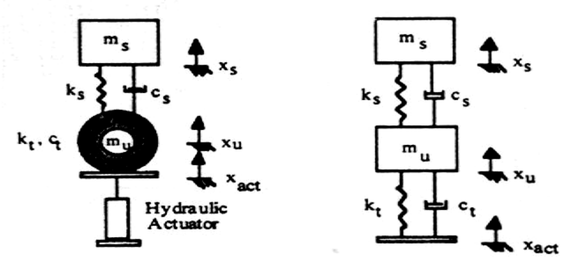

Fig. 1 Vehicle durability simulator schematic

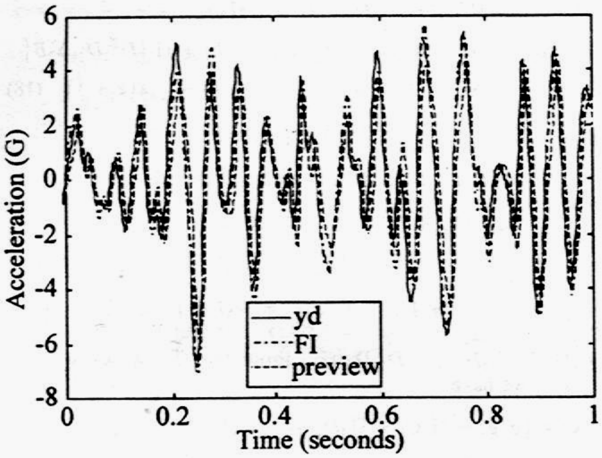

Fig. 2 Tracking results—full information versus preview (continuous-time)

$$F_1(k, 0) = \delta[I + P(k+1)\beta]^{-1}P(k+1)M + N \quad (24)$$

$$F_2(k) = \delta[I + P(k+1)\beta]^{-1}[F_1(k+1, N_{la} - 1) + F_2(k+1)A_w]$$

$$F_2(N) = 0 \quad (25)$$

Again, if we assume $z = C_1x$, then $\alpha = A$, $\beta = B_2B_2^T - \gamma^{-2}B_1B_1^T$, $\chi = C_1^TC_1$, $\delta = A^T$, $M = B_p$ and $N = C_1^TD_{1p}$. Therefore, Eq. (23) reduces to the standard discrete-time game Riccati Equation (i.e., $P(k) = C_1^TC_1 + A^T[I + P(k+1)(B_2B_2^T - \gamma^{-2}B_1B_1^T)]^{-1}P(k+1)A$). When $\gamma \to \infty$, Eqs. (23)–(25) again reduce to LQ-preview control results. The proofs are similar to those of Facts 3.1 and 3.2 and the details are omitted here. We now summarize the results of Sections 4 and 5 in the following theorem.

Theorem 5.1 For the discrete linear time-invariant plant shown in Eq. (16), if (i) (A, B_1) is controllable and (C_1, A) is observable; (ii) (A, B_2) is stabilizable; and (iii) $H \equiv \begin{bmatrix} \alpha & -\beta \\ \chi & \alpha^T \end{bmatrix} \in dom(Ric)$, an admissible H_∞ preview control law is $u(k) = g_1x(k) + g_2w_p(k) + g_3[I + P(k+1)\beta]^{-1} \cdot \{P(k+1)\alpha x(k) + P(k+1)Mw_p(k) + \sum_{n=0}^{N_{la}-1} F_1(k+1, n)w_p(k+1+n) + F_2(k+1)w_p(k+1+N_{la})\}$. The control gain matrices P, F_1, and F_2 are governed by Eqs. (23)–(25), where matrices α, β, χ, δ, M, and N are defined in Eq. (8) (with modifications as noted in the remark below Eq. (21)), and g_1, g_2 and g_3 are defined in Eq. (18).

When w_p is a scalar, the steady-state solutions of Eqs. (23)–(25) are obtained as follows: the feedback matrix P is first obtained from the MATLAB command $P = dlqr(\alpha, \beta^{1/2}, \chi, I)$, where the infinite-horizon version of Eq. (23) first needs to be rewritten as $P = \chi + \alpha^TP\alpha - \alpha^TP\beta^{1/2}[I + \beta^{1/2}P\beta^{1/2}]^{-1}\beta^{1/2}P\alpha$ by using the matrix inversion lemma (Ogata, 1995). By defining $A_c \equiv \delta(I + P\beta)^{-1}$, the preview gains are $F_1(n) = A_c^n(A_cPM + N)$ and $F_2 = [I - A_cA_w]^{-1}A_c^{N_{la}}(A_cPM + N)$, respectively.

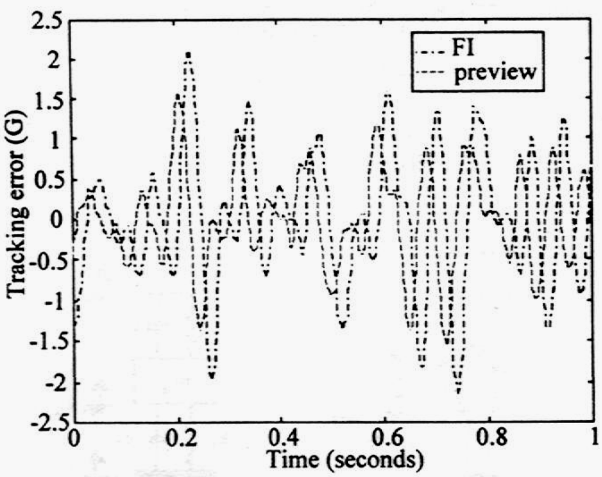

Fig. 3 Tracking error—full information versus preview (continuous-time)

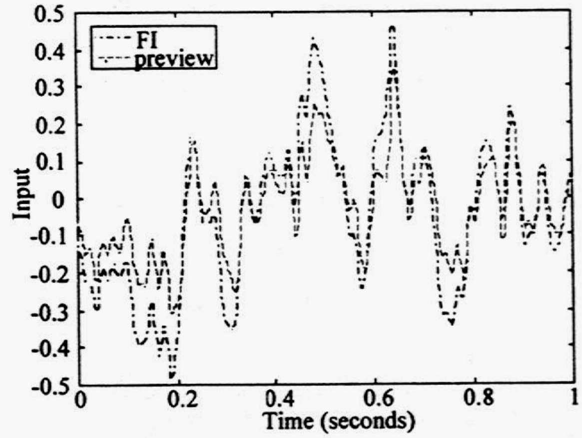

Fig. 4 Input signal—full information versus preview (continuous-time)

6 Application Example: Automotive Durability Test-Rig

In this simulation study, the plant to be controlled is the automotive durability test-rig shown in Fig. 1. The dynamics of this test rig was described in a previous paper (Mianzo and Peng, 1997). The objective of the controller is to manipulate the actuator displacement so that the wheel axle acceleration follows the acceleration profile measured on test tracks as closely as possible. By reproducing the axle acceleration repeatedly, the durability of the vehicle suspension can be assessed more efficiently in a laboratory. The plant dynamics was identified to be

$$\dot{\underline{x}} = \begin{bmatrix} 587.6 & 376.9 & -603.1 \\ -682.2 & -229.5 & 1684.4 \\ 90.2 & -144.3 & -1090.3 \end{bmatrix} \underline{x} + \begin{bmatrix} 2466.2 \\ -5407.3 \\ 2850.8 \end{bmatrix} u$$

$$= Ax + B_2u$$

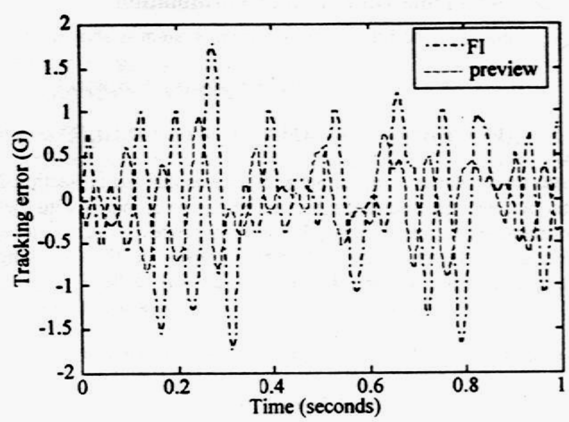

Fig. 5 Tracking error—full information versus preview (discrete-time)

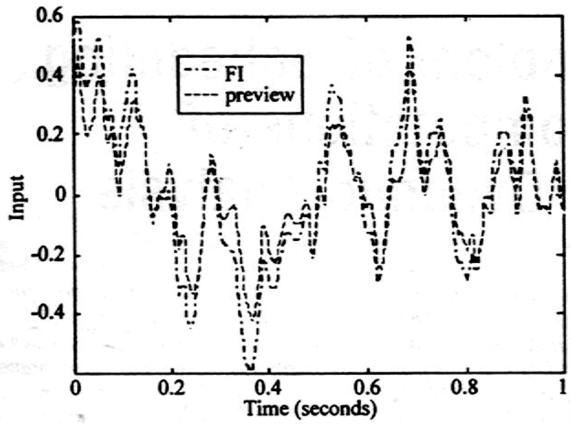

Fig. 6 Input signal—full information versus preview (discrete-time)

$$y = [1 \quad 0 \quad 0]\underline{x} + 12.04u = C_2x + D_{22}u \tag{26}$$

After analyzing several possible uncertainties (tire pressure, actuator aging, etc.), it was determined that $B_1 = [1 \ \ -2 \ \ 1]^T$. For tracking problems, $w_p = y_d$ (the desired axle acceleration). Assuming $z = \rho(y - w_p)$ where ρ is a weighting factor, we have $C_1 = [\rho \ 0 \ 0]$, $D_{11} = 0$, $D_{12} = \rho \cdot D_{22}$, $D_{1p} = -\rho$ and $B_p = [0 \ 0 \ 0]^T$. Therefore, $\Delta_1 = \Pi_1 = -(I + D_{12}^T D_{12})^{-1}$, $\Delta_2 = \Pi_2 = \gamma^{-2}$, $\Pi_3 = C_1^T - C_1^T D_{12}(I + D_{12}^T D_{12})^{-1} D_{12}^T$, $g_1 = -(I + D_{12}^T D_{12})^{-1} D_{12}^T C_1$, $g_2 = -(I + D_{12}^T D_{12})^{-1} D_{12}^T D_{1p}$, $g_3 = -(I + D_{12}^T D_{12})^{-1} B_2^T$, $g_4 = g_5 = 0$, and $g_6 = \gamma^{-2} B_1^T$.

The infinite-horizon preview control law was implemented, and the preview time was chosen to be 20 msec. It can be seen from Figs. 2–4 ($\gamma = 1.5$, $\rho = 6$, no plant uncertainty) that while both control laws provide reasonable tracking results (Fig. 2), the preview control action further reduces the tracking error compared with the full-information (no-preview) case (Fig. 3). A phase-lead contributed by the preview law is obvious from the plots. Another important fact is that the performance improvement is achieved under reduced control effort (Fig. 4), which suggests a more clever use of control resource by the preview control law.

When we apply the discrete preview control described in Section 5, similar improvement was obtained. In the discrete-time simulations, we use a slightly longer preview time (30 msec), which corresponds to a preview step size of 15 (sampling time = 2 ms). The discrete-time preview algorithm again achieves improved tracking performance while using reduced control effort (Figs. 5–6, $\gamma = 1.5$, $\rho = 5$, no plant uncertainty).

7 Conclusions

A Hamiltonian-based formulation is developed to solve both the continuous and discrete-time LQ and H_∞ preview control algorithms. The disturbance signal is divided into previewable (e.g., desired trajectory) and nonpreviewable (e.g., plant uncertainty) parts. When the previewable disturbance is available in a finite preview window, the preview control laws consist of the standard LQ or H_∞ feedback control plus two preview terms. When future information is not available, the control algorithms reduce to standard full information LQ and H_∞ algorithms. Since the feedback control part is identical to their respective LQ and H_∞ feedback-only designs, the preview control algorithm does not change their stability/robustness characteristics. Both continuous and discrete-time simulation results show that preview control laws improve tracking performance while using reduced control effort.

References

Anderson, B. D. O., and Moore, J. B., 1990, *Optimal Control—Linear Quadratic Methods*, Prentice-Hall.

Basar, T., and Bernhard, P., 1995, *H_∞-Optimal Control and Related Minimax Design Problems—A Dynamic Game Approach*, 2nd edition, Birkhauser.

Doyle, J., Glover, K., Khargonekar, P., and Francis, B., 1989, "State-Space Solutions to Standard H_2 and H_∞ Control Problems," *IEEE Transactions on Automatic Control*, Vol. 34, pp. 831–847.

Grimble, M., 1988, "Two-degrees of Freedom Feedback and Feedforward Optimal Control of Multivariable Stochastic Systems," *Automatica*, Vol. 24, pp. 809–817.

Hara, S., and Sugie, T., 1988, "Independent Parameterization of Two-Degree-of-Freedom Compensators in General Robust Tracking Systems," *IEEE Transactions on Automatic Control*, Vol. AC-33, Jan., pp. 59–67.

Ma, W., and Peng, H., 1996, "Solutions of Two-Player Differential Games with Preview Information Structure," Submitted to the ASME Journal of Dynamic Systems, Measurement, and Control, Dec. 1996.

Mianzo, L., and Peng, H., 1997, "LQ and H_∞ Preview Control for a Durability Simulator," *Proceedings of the American Control Conference*, Albuquerque, NM, pp. 699–703.

Moran, A., Nagai, M., and Hayase, M., 1996, "Design of Active Suspensions with H . Preview Control," *Proceedings of International Symposium on Advanced Vehicle Control*, Aachen University of Technology, pp. 215–232.

Ogata, K., 1995, *Discrete-Time Control Systems*, Prentice-Hall, Englewood Cliffs, NJ.

Peng, H., and Tomizuka, M., 1993, "Preview Control for Vehicle Lateral Guidance in Highway Automation," ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 115, No. 4, Dec., pp. 679–686.

Soroka, E., and Shaked, U., 1986, "A Decoupling Approach to the Design of the Two-Degree-of-Freedom Tracking Control Systems," *Proceedings of the 25th Conference on Decision and Control*, Athens, Greece, Dec.

Tomizuka, M., and Whitney, D. E., 1975, "Optimal Discrete Finite Preview Problems (Why and How is Future Information Important?)" ASME Journal of Dynamic Systems, Measurement, and Control, Dec., pp. 319–325.

Yaesh, I., and Shaked, U., 1991, "Two-Degree-of-Freedom H_∞ Optimization of Multivariable Feedback Systems," *IEEE Transactions on Automatic Control*, Vol. 36, No. 11, Nov., pp. 1272–1276.

Zhou, K., and Doyle, J. C., 1998, *Essentials of Robust Control*, Prentice-Hall.