A Unified Framework for LQ and H_∞ Preview Control Algorithms

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Abstract

A general framework to solve both the continuous and discrete-time LQ and H_∞ preview control algorithms is presented in this paper. The obtained preview control form consists of a full-state feedback term which is identical to well known LQ and H_∞ results. The preview terms utilizes previewable disturbance signals to improve tracking or regulation performance. Simulation results of the tracking control of a durability test rig are presented to illustrate the effectiveness of the proposed preview control laws.

1. Introduction

Much work has focused on the H_∞ feedback control problem, since the landmark state-space solution was presented in [1]. A controller that optimizes the H_∞ norm was found to exist if and only if the stabilizing solutions of two algebraic Riccati equations satisfy three inequality conditions (see [1]). Although it is generally accepted that feedforward control can greatly enhance performance, most work in the H_∞ control area focuses on the use of single-degree-of-freedom (SDOF), i.e., feedback only, controller design. In [2], as well as several earlier works, [3], [4], and [5], the two-degree of freedom (2DOF) H_∞ control problems were investigated. Most of these formulations often increase the order of the controller structure, which is a major drawback. For example, an interesting H_∞ spectral factorization approach was proposed in [2] that, although elegant, created a feedforward control that is of the combined order of feedback controller and the augmented plant.

It has been shown that preview control can improve performance when future information about the desired output or exogenous disturbance is available. The LQ based preview control formulation is well established. In an early derivation, [6], the optimal preview control signal was found to consist of three control terms: one feedback and two feedforward terms. These two feedforward terms consist of the preview signal inside the preview window (a convolution term) and outside the preview window (a "kick" term). In [7], the kick term is neglected, resulting in a simpler preview control algorithm. These LQ based preview control algorithms have been applied to a wide range of applications (see [7], [8]) with significant success.

Preview control algorithms based on the H_∞ norm have gained increased interest recently. In [9] and [10], a derivation is proposed based on game theories. The Riccati Equation is then modified accordingly and the feedforward control law is assumed unchanged from the LQ-preview control formulation. A detailed derivation was not presented. In [11], the H_∞ preview control formulation is based on stored disturbances and is given based on finite-dimensional operations. The stored disturbances describes the perturbations of the preview control systems and allows for the derivation of the control law. In [12], a two-player game theory approach is applied to define the preview control law. Here, one player represents the control and the other player represents the exogenous disturbance. Under this approach, the signals of one of the players are generated in future time and previewed by the other (advantaged) player. In [13], which is the precursor to this paper, a different approach is taken. A Hamiltonian based formulation was developed to allow for the simultaneous design of feedback, feedforward, and preview control components. In this paper, the approach of [13] is generalized to a framework suitable for the continuous and discrete-time, LQ and H_∞, and tracking and regulation problems (all the 8 combinations).

2. Continuous Hamiltonian Formulation

The continuous linear time invariant system studied in this paper is assumed to have the following standard form:

$$\begin{aligned}\dot{x} &= Ax + B_1 w + B_2 u + B_p w_p \\ z &= C_1 x + D_{11} w + D_{12} u + D_{1p} w_p \\ y &= C_2 x + D_{21} w + D_{22} u + D_{2p} w_p\end{aligned} \tag{1}$$

where x is the state vector, u is the control, w is the non-previewable and w_p is the previewable disturbance, y is the measured outputs, and z is the set of performance variables. These variables are stated in a form most frequently used in the H_∞ literature. Due to the fact that we are developing full-state feedback schemes, the output vector y is not used in this paper. However, we choose to keep the standard notations so that the results from this paper can be compared with existing H_∞ results more easily. The performance variable z can be properly defined for tracking and regulation problems. For preview control problems, w_p is the desired trajectory for the tracking problem, and previewable disturbance for the regulation case. The nonpreviewable disturbance w has the standard definition and could include model uncertainty, measurement noise and unknown disturbances. The cost function to be minimized by the control vector is assumed to have the following form:

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$$J=\frac{1}{2}\int_0^{t_f}[z^Tz+u^Tu-\gamma^2w^Tw]dt \qquad (2)$$

where for the H2 case, $\gamma\to\infty$. The Hamiltonian is then

$$H(x,\lambda,t)=\frac{1}{2}\left[z^Tz+u^Tu-\gamma^2w^Tw\right] + \lambda^T(Ax+B_1w+B_2u+B_pw_p) \qquad (3)$$

substituting Eq. (1) into (3),

$$H(x,\lambda,t)=\frac{1}{2}[(C_1x+D_{11}w+D_{12}u+D_{1p}w_p)^T \cdot(C_1x+D_{11}w+D_{12}u+D_{1p}w_p)+u^Tu-\gamma^2w^Tw] + \lambda^T(Ax+B_1w+B_2u+B_pw_p) \qquad (4)$$

The optimal control can be obtained from $\frac{\partial H}{\partial u}=0$ and $\frac{\partial H}{\partial w}=0$. Therefore,

$$\begin{aligned} u=&\Pi_1(D_{12}^TC_1+D_{12}^TD_{11}\Delta_2D_{11}^TC_1)x \\ &+\Pi_1(D_{12}^TD_{11}\Delta_2D_{11}^TD_{1p}+D_{12}^TD_{1p})w_p \qquad (5)\\ &+\Pi_1(D_{12}^TD_{11}\Delta_2B_1^T+B_2^T)\lambda\equiv g_1x+g_2w_p+g_3\lambda \end{aligned}$$

$$\begin{aligned} w=&\Pi_2(D_{11}^TC_1+D_{11}^TD_{12}\Delta_1D_{12}^TC_1)x \\ &+\Pi_2(D_{11}^TD_{12}\Delta_1D_{12}^TD_{1p}+D_{11}^TD_{1p})w_p \qquad (6)\\ &+\Pi_2(D_{11}^TD_{12}\Delta_1B_2^T+B_1^T)\lambda\equiv g_4x+g_5w_p+g_6\lambda \end{aligned}$$

where

$$\begin{aligned} \Delta_1&=-(I+D_{12}^TD_{12})^{-1}\\ \Pi_1&=-(I+D_{12}^TD_{11}\Delta_2D_{11}^TD_{12}+D_{12}^TD_{12})^{-1}\\ \Delta_2&=(\gamma^2-D_{11}^TD_{11})^{-1}\\ \Pi_2&=(\gamma^2-D_{11}^TD_{12}\Delta_1D_{12}^TD_{11}-D_{11}^TD_{11})^{-1} \end{aligned}$$

Substitute Eqs.(5) and (6) into (1), we have

$$\begin{aligned} \dot{x}=&Ax+B_1\Pi_2(D_{11}^TC_1+D_{11}^TD_{12}\Delta_1D_{12}^TC_1)x \qquad (7)\\ &+B_2\Pi_1(D_{12}^TC_1+D_{12}^TD_{11}\Delta_2D_{11}^TC_1)x\\ &+B_1\Pi_2(D_{11}^TD_{12}\Delta_1D_{12}^TD_{1p}+D_{11}^TD_{1p})w_p\\ &+B_2\Pi_1(D_{12}^TD_{11}\Delta_2D_{11}^TD_{1p}+D_{12}^TD_{1p})w_p\\ &+B_1\Pi_2(D_{11}^TD_{12}\Delta_1B_2^T+B_1^T)\lambda\\ &+B_2\Pi_1(D_{12}^TD_{11}\Delta_2B_1^T+B_2^T)\lambda \end{aligned}$$

Finally, use the fact that $-\dot{\lambda}^T=\frac{\partial H}{\partial x}$, we have

$$\begin{aligned} \dot{\lambda}=&\left[\gamma^2g_4^Tg_4-g_1^Tg_1-\Pi_3C_1-\Pi_3D_{11}g_4-\Pi_3D_{12}g_1\right]x\\ &+\left[\gamma^2g_4^Tg_6-A^T-\Pi_3D_{11}g_6-\Pi_3D_{12}g_3-g_1^Tg_3-g_1^TB_2^T-g_4^TB_1^T\right]\lambda\\ &+\left[\gamma^2g_4^Tg_5-\Pi_3D_{11}g_5-\Pi_3D_{12}g_2-\Pi_3D_{1p}-g_1^Tg_2\right]w_p \qquad (8) \end{aligned}$$

where

$\Pi_3=C_1^T+(C_1^TD_{11}+C_1^TD_{12}\Delta_1^TD_{12}^TD_{11})\Pi_2^TD_{11}^T + (C_1^TD_{12}+C_1^TD_{11}\Delta_2^TD_{11}^TD_{12})\Pi_1^TD_{12}^T$. The state and costate equation can then be written in a standard Hamiltonian matrix form:

$$\begin{bmatrix}\dot{x}\\ \dot{\lambda}\end{bmatrix}=\begin{bmatrix}\alpha & -\beta\\ \chi & \delta\end{bmatrix}\begin{bmatrix}x\\ \lambda\end{bmatrix}+\begin{bmatrix}M\\ N\end{bmatrix}w_p \qquad (9)$$

where

$$\begin{aligned} \alpha=&A+B_1\Pi_2(D_{11}^TC_1+D_{11}^TD_{12}\Delta_1D_{12}^TC_1)\\ &+B_2\Pi_1(D_{12}^TC_1+D_{12}^TD_{11}\Delta_2D_{11}^TC_1)\\ \beta=&-B_1\Pi_2(D_{11}^TD_{12}\Delta_1B_2^T+B_1^T)-B_2\Pi_1(D_{12}^TD_{11}\Delta_2B_1^T+B_2^T)\\ \chi=&\gamma^2g_4^Tg_4-g_1^Tg_1-\Pi_3C_1-\Pi_3D_{11}g_4-\Pi_3D_{12}g_1\\ \delta=&\gamma^2g_4^Tg_6-A^T-\Pi_3D_{11}g_6-\Pi_3D_{12}g_3-g_1^Tg_3-g_1^TB_2^T-g_4^TB_1^T\\ M=&B_1\Pi_2(D_{11}^TD_{12}\Delta_1D_{12}^TD_{1p}+D_{11}^TD_{1p})\\ &+B_2\Pi_1(D_{12}^TD_{11}\Delta_2D_{11}^TD_{1p}+D_{12}^TD_{1p})+B_p \qquad (10)\\ N=&\gamma^2g_4^Tg_5-\Pi_3D_{11}g_5-\Pi_3D_{12}g_2-\Pi_3D_{1p}-g_1^Tg_2 \end{aligned}$$

It is important to note that after tedious but straightforward algebraic manipulation, it can be shown that $\delta=-\alpha^T$. If we make the standard assumptions (e.g. $z=C_1x$), the Hamiltonian matrix of Eq.(9) reduces to the one commonly seen in the H_∞ literature. We choose not to make those assumptions so that the results can be used in more general cases.

3. LQ And H∞ Continuous-Time Preview Control Algorithms

The linear differential equations shown in Eq.(10) form a two point boundary value problem with mixed boundary conditions. The mixed value boundary conditions make the problem difficult to solve. One possible option is to assume a solution, and then find the constraints imposed by the optimal condition for this particular control form. When the "disturbance" term is previewable within a preview window, i.e., at time t, the signal $w_p(\tau)$, $\tau\in[t,t+t_{la}]$ is known, one possible form for the costate dynamics, inspired by previous H2 results ([6]) is

$$\lambda=Px+\int_0^{t_{la}}F_1(t,\tau)w_p(t+\tau)d\tau+F_2(t)w_p(t+t_{la}) \qquad (12)$$

where the first term represents the homogeneous part (full-state feedback) while the second and third terms are the assumed solutions for the nonhomogeneous part (preview control). When there is no preview, the problem reduces to a homogenous two point boundary value problem. Differentiating both sides of Eq.(12) w.r.t. time, we obtain

$$\begin{aligned} \frac{d\lambda}{dt}=&\frac{dP}{dt}x+P\frac{dx}{dt}+\int_0^{t_{la}}F_1(t,\tau)\frac{d}{dt}w_p(t+\tau)d\tau \qquad (13)\\ &+\int_0^{t_{la}}\frac{d}{dt}F_1(t,\tau)w_p(t+\tau)d\tau+\dot{F}_2(t)w_p(t+t_{la})+F_2(t)\dot{w}_p(t+t_{la})\\ =&\chi x+\delta(Px+\int_0^{t_{la}}F_1(t,\tau)w_p(t+\tau)d\tau+F_2(t)w_p(t+t_{la}))+Nw_p(t) \end{aligned}$$

Or,

$$\begin{aligned} &\frac{dP}{dt}x+P[\alpha x-\beta(Px+\int_0^{t_{la}}F_1(t,\tau)w_p(t+\tau)d\tau \qquad (14)\\ &+F_2(t)w_p(t+t_{la}))+Mw_p(t)]+\int_0^{t_{la}}F_1(t,\tau)\frac{d}{dt}w_p(t+\tau)d\tau\\ &+\int_0^{t_{la}}\frac{d}{dt}F_1(t,\tau)w_p(t+\tau)d\tau+\dot{F}_2(t)w_p(t+t_{la})+F_2(t)\dot{w}_p(t+t_{la})\\ &=\chi x+\delta(Px+\int_0^{t_{la}}F_1(t,\tau)w_p(t+\tau)d\tau+F_2(t)w_p(t+t_{la}))+Nw_p(t) \end{aligned}$$

To simplify Eq.(14), we need to make one assumption about the disturbance signal outside of the preview window. The assumption commonly used in the LQ preview control algorithms [13] is:

$$\frac{dw_p}{d\tau} = A_w w_p(\tau) \quad \tau \geq t + t_{la} \tag{15}$$

Usually, A_w is set to 0, i.e., the disturbance is assumed to remain constant outside of the preview window.
Apply Eq.(15), and use the following fact:

$$\int_0^{t_{la}} F_1(t,\tau)\frac{d}{dt}w_p(t+\tau)d\tau = \int_0^{t_{la}} F_1(t,\tau)\frac{d}{d\tau}w_p(t+\tau)d\tau$$
$$= F_1(t,\tau)w_p(t+\tau)|_0^{t_{la}} - \int_0^{t_{la}} \frac{d}{d\tau}F_1(t,\tau)w_p(t+\tau)d\tau \tag{16}$$

Eq. (15) can be rewritten as

$$\frac{dP}{dt}x + P\alpha x - P\beta Px - P\beta\int_0^{t_{la}} F_1(t,\tau)w_p(t+\tau)d\tau$$
$$-P\beta F_2(t)w_p(t+t_{la})) + PMw_p(t)$$
$$+F_1(t,\tau)w_p(t+\tau)|_0^{t_{la}} - \int_0^{t_{la}} \frac{d}{d\tau}F_1(t,\tau)w_p(t+\tau)d\tau$$
$$+\int_0^{t_{la}} \frac{d}{dt}F_1(t,\tau)w_p(t+\tau)d\tau + \dot{F}_2(t)w_p(t+t_{la})$$
$$+F_2(t)A_w w_p(t+t_{la}) = \chi x + \delta Px \tag{17}$$
$$+\delta\int_0^{t_{la}} F_1(t,\tau)w_p(t+\tau)d\tau + \delta F_2(t)w_p(t+t_{la}) + Nw_p(t)$$

In order to have Eq.(17) satisfied under all situations, we have

$$\frac{dP}{dt} + P\alpha - P\beta P - \chi - \delta P = 0 \tag{18}$$
$$P(t_f) = 0$$
$$-P\beta F_1(t,\tau) - \frac{\partial}{\partial\tau}F_1(t,\tau) + \frac{\partial}{\partial t}F_1(t,\tau) - \delta F_1(t,\tau) = 0 \tag{19}$$
$$F_1(t,0) = PM - N \tag{20}$$
$$-P\beta F_2(t) + F_1(t,t_{la}) + \dot{F}_2(t) + F_2(t)A_w - \delta F_2(t) = 0 \tag{21}$$
$$F_2(t_f) = 0$$

which are obtained by grouping similar terms in Eq.(17). The boundary conditions for Eqs.(18) and (21) are obtained by using the fact that $\lambda(t_f) = \partial J^*(t_f)/\partial x = 0$, where J^* is the optimal cost. When $z = C_1 x$ (i.e., $D_{11} = D_{12} = 0$), $\alpha = A$, $\beta = B_2B_2^T - \gamma^{-2}B_1B_1^T$, $\chi = -C_1^TC_1$, $\delta = -A^T$, $M = B_p$ and $N = -C_1^TD_{1p}$. Therefore, Eq.(18) reduces to the standard Riccati Equation

$$\dot{P} + PA - P(B_2B_2^T - \gamma^{-2}B_1B_1^T)P + C_1^TC_1 + A^TP = 0.$$

The optimal preview control signal, which is obtained by solving Eqs.(18)-(21), and plugging the results to Eq.(5) and (12), consists of a feedback and two feedforward terms. The feedback term is exactly the same as that of the standard LQ or H_∞ problem. This is because the Riccati equation is exactly the same as that of the non-preview problems. The first feedforward term is a convoluted integral of the previewable signal inside the preview window. The second feedforward term is a kick term, which deals with the previewable signal outside the preview window. Since the feedback part of the preview control algorithm is exactly the same as the standard LQ and H_∞ problem, stability of the closed-loop system is guaranteed.

4. Discrete Hamiltonian Formulation

The discrete linear time invariant system is assumed to have the following form:

$$x(k+1) = Ax(k) + B_1w(k) + B_2u(k) + B_pw_p(k)$$
$$z(k) = C_1x(k) + D_{11}w(k) + D_{12}u(k) + D_{1p}w_p(k) \tag{22}$$
$$y(k) = C_2x(k) + D_{21}w(k) + D_{22}u(k) + D_{2p}w_p(k)$$

where *x(k)* is the state vector, *u(k)* is the control, *w(k)* is the non-previewable disturbance vector, $w_p(k)$ is the previewable disturbance, *y(k)* is measured outputs, and *z(k)* is the performance vector. The cost function to be minimized is assumed to be

$$J = \frac{1}{2}\sum_0^{N_f}[z^Tz + u^Tu - \gamma^2w^Tw] \tag{23}$$

The Hamiltonian is thus

$$H(x,\lambda,k) = \frac{1}{2}\left[z(k)^Tz(k) + u(k)^Tu(k) - \gamma^2w^T(k)w(k)\right]$$
$$+\lambda(k+1)^T(Ax(k) + B_1w(k) + B_2u(k) + B_pw_p(k)) \tag{24}$$

Differentiating Eq.(24) with respect to u and w, the optimal condition can be achieved if we have

$$D_{12}^TC_1x(k) + D_{12}^TD_{11}w(k) + D_{12}^TD_{12}u(k)$$
$$+D_{12}^TD_{1p}w_p(k) + u(k) + B_2^T\lambda(k+1) = 0 \tag{25}$$
$$D_{11}^TC_1x(k) + D_{11}^TD_{11}w(k) + D_{11}^TD_{12}u(k)$$
$$+D_{11}^TD_{1p}w_p(k) - \gamma^2w(k) + B_1^T\lambda(k+1) = 0 \tag{26}$$

Or, after some rearrangement,

$$u(k) = \Pi_1(D_{12}^TC_1 + D_{12}^TD_{11}\Delta_2D_{11}^TC_1)x(k)$$
$$+\Pi_1(D_{12}^TD_{11}\Delta_2D_{11}^TD_{1p} + D_{12}^TD_{1p})w_p(k)$$
$$+\Pi_1(D_{12}^TD_{11}\Delta_2B_1^T + B_2^T)\lambda(k+1)$$
$$\equiv g_1x(k) + g_2w_p(k) + g_3\lambda(k+1) \tag{27}$$
$$w(k) = \Pi_2(D_{11}^TC_1 + D_{11}^TD_{12}\Delta_1D_{12}^TC_1)x(k)$$
$$+\Pi_2(D_{11}^TD_{12}\Delta_1D_{12}^TD_{1p} + D_{11}^TD_{1p})w_p(k)$$
$$+\Pi_2(B_1^T + D_{11}^TD_{12}\Delta_1B_2^T)\lambda(k+1)$$
$$\equiv g_4x(k) + g_5w_p(k) + g_6\lambda(k+1) \tag{28}$$

where $\Delta_1 = -(I + D_{12}^TD_{12})^{-1}$, $\Delta_2 = (\gamma^2 - D_{11}^TD_{11})^{-1}$, $\Pi_1 = -(I + D_{12}^TD_{11}\Delta_2D_{11}^TD_{12} + D_{12}^TD_{12})^{-1}$, and $\Pi_2 = (\gamma^2 - D_{11}^TD_{12}\Delta_1D_{12}^TD_{11} - D_{11}^TD_{11})^{-1}$. From the fact $\lambda^T(k) = \frac{\partial H}{\partial x(k)}$, Eq.(24) can be differentiated with respect to x to get the costate equation

$$\lambda(k) = \left[g_1^Tg_1 + \Pi_3C_1 + \Pi_3D_{11}g_4 + \Pi_3D_{12}g_1 - \gamma^2g_4^Tg_4\right]x(k)$$
$$+\left[A^T + \Pi_3D_{11}g_6 + \Pi_3D_{12}g_3 + g_1^Tg_3 + g_1^TB_2^T + g_4^TB_1^T - \gamma^2g_4^Tg_6\right]\lambda(k+1)$$
$$+\left[\Pi_3D_{11}g_5 + \Pi_3D_{12}g_2 + \Pi_3D_{1p} + g_1^Tg_2 - \gamma^2g_4^Tg_5\right]w_p(k) \tag{29}$$

where

$$\Pi_3 = C_1^T + (C_1^TD_{11} + C_1^TD_{12}\Delta_1^TD_{12}^TD_{11})\Pi_2^TD_{11}^T$$
$$+(C_1^TD_{12} + C_1^TD_{11}\Delta_2^TD_{11}^TD_{12})\Pi_1^TD_{12}^T.$$

Finally, by plugging-in Eqs.(29) into Eq.(22), we have

$$x(k+1) = Ax(k) + B_1\Pi_2(D_{11}^TC_1 + D_{11}^TD_{12}\Delta_1D_{12}^TC_1)x(k)$$

$$+B_2\Pi_1(D_{12}^T C_1 + D_{12}^T D_{11}\Delta_2 D_{11}^T C_1)x(k)$$
$$+B_1\Pi_2(D_{11}^T D_{12}\Delta_1 D_{12}^T D_{1p} + D_{11}^T D_{1p})w_p(k)$$
$$+B_2\Pi_1(D_{12}^T D_{11}\Delta_2 D_{11}^T D_{1p} + D_{12}^T D_{1p})w_p(k) + B_p w_p(k)$$
$$+B_1\Pi_2(B_1{}^T + D_{11}^T D_{12}\Delta_1 B_2^T)\lambda(k+1)$$
$$+B_2\Pi_1(D_{12}^T D_{11}\Delta_2 B_1{}^T + B_2^T)\lambda(k+1) \qquad (30)$$

Eqs.(29) and (30) can be written in the matrix form

$$\begin{bmatrix} x(k+1) \\ \lambda(k) \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ \chi & \delta \end{bmatrix} \begin{bmatrix} x(k) \\ \lambda(k+1) \end{bmatrix} + \begin{bmatrix} M \\ N \end{bmatrix} w_p(k) \qquad (31)$$

The values for α, β, χ, δ, M and N are almost identical to their counterparts defined in Eq.(10), except that χ, δ and N are defined with an extra minus sign (i.e., $\chi_{discrete} = -\chi_{continuous}$, etc.) After tedious algebraic manipulation, it can be shown that $\delta = \alpha^T$.

5. LQ And H_∞ Discrete-Time Preview Control Algorithm

Eq. (31) forms a discrete two point boundary value problem, with mixed boundary conditions. As in the continuous-time problem, a solution for the costate dynamics is assumed:

$$\lambda(k) = P(k)x(k) + \sum_{n=0}^{N_{la}-1} F_1(k,n)p(k+n) + F_2(k)p(k+N_{la}) \qquad (32)$$

Therefore,

$$\lambda(k+1) = P(k+1)x(k+1) + \sum_{n=0}^{N_{la}-1} F_1(k+1,n)p(k+n+1)$$
$$+F_2(k+1)p(k+N_{la}+1) \qquad (33)$$
$$= P(k+1)\left[\alpha x(k) - \beta\lambda(k+1) + M w_p(k)\right]$$
$$+ \sum_{n=0}^{N_{la}-1} F_1(k+1,n)p(k+n+1) + F_2(k+1)p(k+N_{la}+1)$$

Therefore,

$$\lambda(k+1) = [I + P(k+1)\beta]^{-1}[P(k+1)\alpha x(k) + P(k+1)Mw_p(k)$$
$$+ \sum_{n=0}^{N_{la}-1} F_1(k+1,n)p(k+n+1) + F_2(k+1)p(k+N_{la}+1)] \qquad (34)$$

From Eqs.(31), (32) and (34)

$$\lambda(k) = P(k)x(k) + \sum_{n=0}^{N_{la}-1} F_1(k,n)p(k+n) + F_2(k)p(k+N_{la})$$
$$= \chi x(k) + \delta[[I + P(k+1)\beta]^{-1}\{P(k+1)\alpha x(k) + P(k+1)Mw_p(k)$$
$$+ \sum_{n=0}^{N_{la}-1} F_1(k+1,n)p(k+n+1) + F_2(k+1)p(k+N_{la}+1)\}] + Nw_p(k) \qquad (35)$$

Similar to the continuous-time case, the dynamics of the disturbance outside of the preview are assumed to be known and are described by $p(k+j+1) = A_w p(k+j)$, $j \ge N_{la}$. Frequently A_w is assumed to be an identity matrix, i.e., disturbance outside of the preview window is assumed to remain constant. Since Eq.(35) needs to be satisfied by all $x(k)$ and $w_p(k)$, the optimal condition is satisfied if we have

$$P(k) = \chi + \delta\left[I + P(k+1)\beta\right]^{-1} P(k+1)\alpha \qquad (36)$$
$$P(N) = 0$$
$$F_1(k,n) = \delta\left[I + P(k+1)\beta\right]^{-1} F_1(k+1,n-1) \qquad (37)$$
$$F_1(k,0) = \delta\left[I + P(k+1)\beta\right]^{-1} P(k+1)M + N \qquad (38)$$
$$F_2(k) = \delta\left[I + P(k+1)\beta\right]^{-1}\left[F_1(k+1,N_{la}-1) + F_2(k+1)A_w\right]$$
$$F_2(N) = 0 \qquad (39)$$

where Eqs.(36)-(39) are obtained by grouping similar terms in Eq.(35). The boundary conditions of Eqs.(36) and (39) are obtained from $J^*(N) = 0$. When $z = C_1 x$ ($D_{11} = D_{12} = 0$), $\alpha = A$, $\beta = B_2 B_2^T - \gamma^{-2} B_1 B_1^T$, $\chi = C_1^T C_1$, $\delta = A^T$, $M = B_p$ and $N = C_1^T D_{1p}$. Therefore, Eq.(36) reduces to the standard discrete-time Riccati Equation (i.e.,

$$P(k) = C_1^T C_1 + A^T\left[I + P(k+1)\left(B_2 B_2^T - \gamma^{-2} B_1 B_1^T\right)\right]^{-1} P(k+1)A.$$

Again, the optimal preview control gains for both H_∞ and LQ problems can be obtained from Eqs.(36)-(39). Due to the fact that we maintain a full information structure and the fact that the problem formulation can be applied to both tracking and regulation problems, these equations appear complicated. We will show in the next section that these equations not only reduce to standard H_∞ and LQ forms under the non-preview case, but also the fact that in specific preview cases, these equations are actually quite simple.

6. Numerical Example: Continuous H_∞ and LQ Tracking Case

The plant to be controlled is the vehicle durability simulator as shown in Figure 1. The dynamics of this test rig have been described in [13]. The objective of the controller is to manipulate the actuator displacement (the control input) so that the axle (unsprung mass) acceleration follows the acceleration profile measured on test tracks as closely as possible.

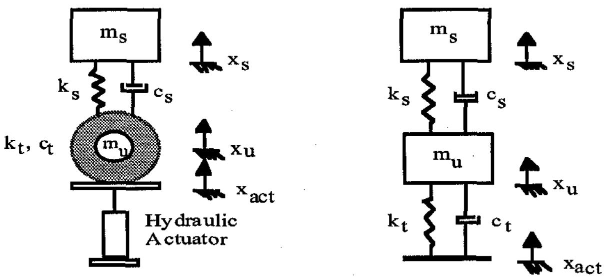

Figure 1. Vehicle Durability Simulator Schematic

Instead of developing a physics-based model, we chose to use system identification techniques to obtain an input/output model from experimental data. The identified plant is 3rd order and was found to be

$$\underline{\dot{x}} = \begin{bmatrix} 587.6 & 376.9 & -603.1 \\ -682.2 & -229.5 & 1684.4 \\ 90.2 & -144.3 & -1090.3 \end{bmatrix} \underline{x} + \begin{bmatrix} 2466.2 \\ -5407.3 \\ 2850.8 \end{bmatrix} u$$
$$= Ax + B_2 u \qquad (40)$$
$$y = [1 \quad 0 \quad 0]\underline{x} + 12.04u \;= C_2 x + D_{22} u$$

After analyzing several possible plant uncertainties (tire pressure, actuator aging, hydraulic pressure variation, Coulomb friction, backlash, and sensor geometric noise), it was determined that a good selection of disturbance matrices will be $B_1 = [1 \;\; 2 \;\; 1]^T$. For tracking problems, $w_p = y_d$, the desired output. Assuming $z = \rho(y - w_p)$ where ρ is a weighting factor, we have $C_1 = [\rho \;\; 0 \;\; 0]$, $D_{11} = 0$, $D_{12} = \rho \cdot D_{22}$, $D_{1p} = -\rho$ and $B_p = [0 \;\; 0 \;\; 0]^T$. Therefore, $\Delta_1 = \Pi_1 = -(I + D_{12}^T D_{12})^{-1}$, $\Delta_2 = \Pi_2 = \gamma^{-2}$, $\Pi_3 = C_1^T - C_1^T D_{12}(I + D_{12}^T D_{12})^{-1} D_{12}^T$, $g_1 = -(I + D_{12}^T D_{12})^{-1} D_{12}^T C_1$, $g_2 = -(I + D_{12}^T D_{12})^{-1} D_{12}^T D_{1p}$, $g_3 = -(I + D_{12}^T D_{12})^{-1} B_2^T$, $g_4 = g_5 = 0$, and $g_6 = \gamma^{-2} B_1^T$.

If we use infinite-horizon solutions, the matrix P can be solved by using the MATLAB command $are(\alpha, \beta, -\chi)$. Once P is known, the feedforward gain matrices can be solved from $F_1(\tau) = e^{-(P\beta+\delta)\tau}[PM - N]$ and $F_2 = (\delta - A_w + P\beta)^{-1} F_1(t_{la})$. The optimal control law is then

$$u(t) = (g_1 + g_3 P)x(t) + g_2 w_p(t) + g_3\left[\int_0^{t_{la}} F_1(\tau) w_p(t+\tau)d\tau + F_2 w_p(t + t_{la})\right] \quad (41)$$

We subsequently perform simulation studies to compare full information control (first two terms of Eq.(41), terminology borrowed from [1]) with preview control (complete implementation of Eq.(41)). The preview time was selected to be 10msec.

Figures 2-4 show the comparison between full information and preview control (H_∞, no plant uncertainty). It can be seen that while both control laws provide reasonable tracking results (Figure 2), preview control action further improves the tracking error compared with the full information control law (see Figure 3). A phase shift (lead) contributed by the preview action can be easily observed from the plot. Another important fact is that this performance improvement is achieved under reduced control effort (Figure 4). The reduction in control effort seems to suggest a more clever usage of control resource is achieved by the preview control compared with the full information control law.

For the H_∞ case, due to the assumption on the existence of unpreviewable disturbances, the weighting on the error term needs to be limited. For the LQ case, a higher penalty weighting on the tracking error can be used, i.e., better nominal tracking performance can be achieved. However, since no unpreviewable disturbance is assumed in the LQ design, it is more vulnerable when such disturbances do exist. This fact is well known, and no simulation results will be presented here. Essentially, for LQ designs, γ is assumed to approach infinity, and all unpreviewable disturbance related terms (i.e., B_1, D_{11}, D_{21}, Δ_2, Π_2, g_4, g_5, and g_6) are dropped. Other than this minor change, all the equations and simulation programs can be applied to both H_∞ and LQ control designs and simulations. It should also be noted that in the LQ case, the results presented in this paper reduce to the well-known LQ preview control algorithms (e.g., [8]) when we let γ to approach infinity.

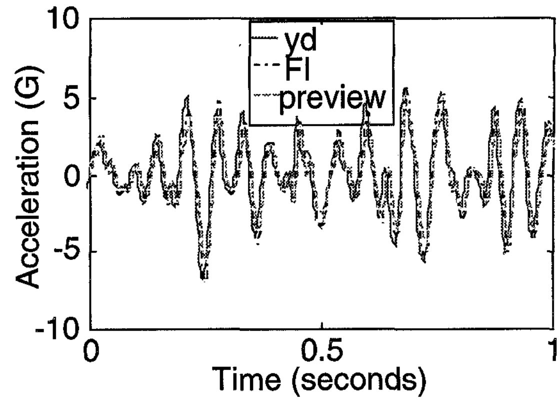

Figure 2 Tracking results--full information vs. preview (continuous-time)

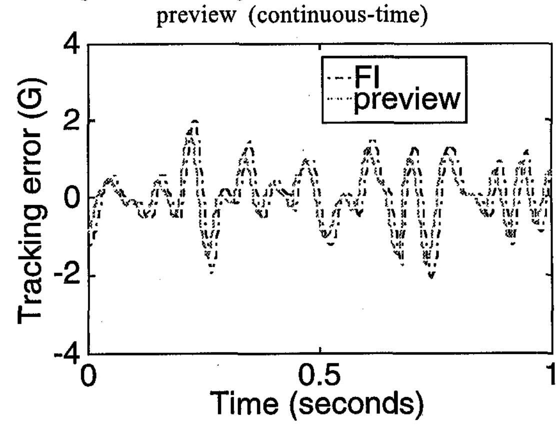

Figure 3 Tracking error-- full information vs. preview (continuous-time)

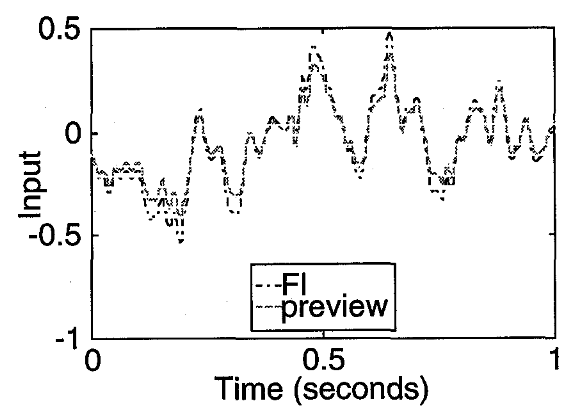

Figure 4 Input signal-- full information vs. preview (continuous-time)

The durability test rig is discretized so that the discrete-time preview control algorithm can be demonstrated. All the Π_i, Δ_i and g_i matrices are identical to their continuous-time counterparts. For the infinite-horizon case, the P matrix can be solved by using the MATLAB command $dlqr(\alpha, \beta^{\frac{1}{2}}, \chi, I)$. The preview matrices are then solved from $F_1(n) = \delta(I + P\beta)^{-n}\left[\delta(I + P\beta)^{-1} PM + N\right]$, and

$F_2 = \left[I - \delta(I + P\beta)^{-1} A_w\right]^{-1} F_1(N_{la} - 1)$. In the following simulations, we use a slightly longer preview time (30msec). The longer preview time is the main reason of the more significant performance improvement. Similar to the continuous-time results, the discrete-time preview control algorithm was found to improve the tracking performance by using a slightly reduced control effort (Figures 5-6).

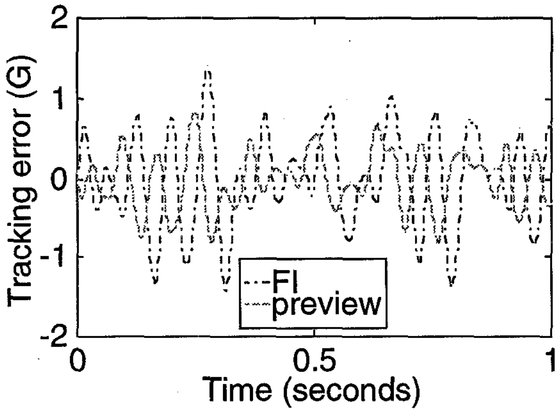

Figure 5 Tracking error-- full information vs. preview (discrete-time)

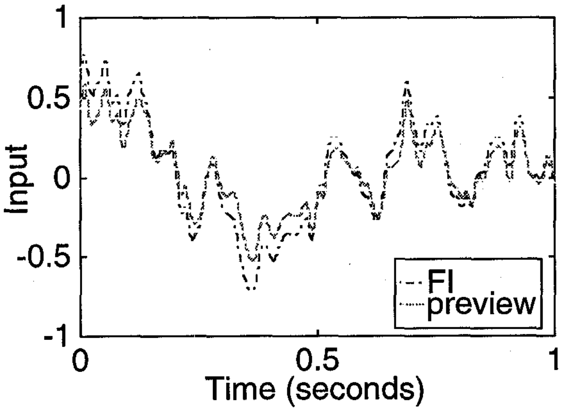

Figure 6 Input signal-- full information vs. preview (discrete-time)

7. Conclusions

A Hamiltonian-based formulation is developed to solve both the continuous and discrete-time LQ and H_∞ preview control algorithms. The "disturbance" signal is divided into previewable (e.g., desired trajectory) and unpreviewable (e.g., plant uncertainty) parts. When future information is not available, it is shown that the control algorithms reduce to standard full information LQ and H_∞ algorithms. Both continuous and discrete-time simulation results on the tracking control of a durability test rig are presented. It is shown that improved tracking can be achieved while the control signal is actually reduced.

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