String Stability Analysis of Adaptive Cruise Controlled Vehicles

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Key Words: Adaptive Cruise Control, String Stability, String Stability Margin, Optimal ACC Design, Traffic Simulator

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ABSTRACT

A framework for string-stability analysis is formulated in this paper. First, uniform string will be analyzed. We will then present analysis results on strings of mixed vehicles. A String-Stability Margin (SSM) index is defined in this paper to give a quantitative measurement of any ACC design. Simulation results using MATLAB and a microscopic traffic simulator will also be given to demonstrate the effectiveness of ACC systems on traffic smoothness.

1. INTRODUCTION

Adaptive Cruise Control (ACC) system has been proposed as an enhancement over existing cruise controllers on ground vehicles. ACC systems control the vehicle speed to follow a driver's set speed closely when no lead vehicle is insight. When a slower leading vehicle is present, the ACC controlled vehicle will follow the lead vehicle at a safe distance. ACC research first began in the 1960's [1], and has received ever-growing attention in the last decade. Their commercial implementation is not possible until recently with significant progresses in sensors, actuators, and other enabling technologies.

Over the last 10 years, many different approaches have been proposed for the design of ACC algorithms. In the earlier works, the focus has been on the performance of the host vehicle. The performance was usually evaluated based on 2-car platoons. The effect of ACC on a string of vehicles has not received much attention. Recently, a 2-step synthesis method was proposed which can be used to design ACC algorithm with guaranteed stability [2]. At the upper level, desired vehicle acceleration is computed based on vehicle range and range rate measurement. At the lower (servo) level, an adaptive control algorithm is designed to ensure the vehicle follows the acceleration command accurately. It is shown that it is possible to include servo-level dynamics in the overall design and string stability can be guaranteed if certain inequality constraints are satisfied.

The so-called "string stability" problem has been studied as early as 1977 [4]. The string-stability ensures that range errors decrease as they propagate along he vehicle stream. It is widely known that when the transfer function from the range error of a vehicle to that of its following vehicle has a magnitude less than 1, string stability is guaranteed [5]. To achieve string stability with constant inter-vehicle spacing, vehicle-to-vehicle communication was shown to be necessary [6]. Yanakiev and Kanellakopoulos [7] used a simple spring-mass-damper system to demonstrate the idea of string stability and show the string-stability criterion for constant time-headway and variable time-headway policies. Swaroop and Hedrick [8] proved, among many other interesting things that if the coupling between two vehicles is weak enough, the controlled system is string stable.

Despite the strides made in the research field, many of the prototype ACC vehicles were still designed without considering string stability. This is mainly due to the fact that human drivers and passengers are used to the level of smoothness produced by human drivers. The early implementation of ACC hardware is usually marked by slow response of actuation and rough distance and speed measurement. In order to produce smooth acceleration/deceleration response, the ACC controllers are usually heavily filtered. Slower response from individual vehicles then results in unstable string response.

In this paper, we plan to demonstrate a detailed stability analysis of a string of vehicles. First, uniform string will be analyzed. Necessary and sufficient conditions for string stability will be presented. We will then present analysis results on strings of mixed vehicles. It will be shown that if we mix ACC vehicles that are designed to be string stable by themselves, the mixed ACC strings will also be string stable. The stability degradation will be shown in the context of the "string stability margin" (SSM) which is an index defined to measure the string stability of a vehicle.

A simulation tool (UMACC) is developed in the University of Michigan to assist the analysis of ACC system. This simulator is divided into four parts: driver model, vehicle/sensor model, ACC model, and the interaction model (see Fig. 1). The driver model consists of a lane change model (for all vehicles) and a car-following model (only for manually controlled vehicles). The parameters of the driver model are obtained from the FOCAS field test data [9]. Details of this model can be found in [3].

Fig 1

A simulation study using ACC Simulator has shown that the vehicle string size increases dramatically with the traffic density (See Fig. 2,3). Because manual vehicles are usually not string stable, the large platoon size in high traffic density will magnify the so-called "slinky effect" and caused traffic jammed. A string stable ACC design is able to reduce this slinky effect and thus improve highway traffic. In this paper, the performance of ACC vehicles will be evaluated by mixing ACC vehicles and manually controlled vehicles using the ACC Simulator. Simulation results using the UMACC Simulator will be presented to highlight the effect of ACC vehicles on the traffic volume and smoothness.

Fig 2,3

The remainder of this paper is organized as follows: the formulation of vehicle-string analysis is presented in Section 2. In Section 3, both uniformed and mixed vehicle strings will be analyzed. The definition of SSM and the SSM calculation of an optimal ACC design will be given in Section 4. Representative simulation results are given in Section 5.

2. PROBLEM FORMULATION

Fig 4

Considering a group of vehicles form a string in dense traffic where no passing occur (Fig. 4) and assuming the operation of each vehicle looks only one vehicle ahead, each vehicle in this string can be modeled as following:

$$x_i = \frac{1}{s} v_i$$
$$v_i = G_i(s) \cdot v_{i-1} \tag{1}$$

where v_i is the velocity of the ith vehicle and G_i represents the car-following algorithm of the ith vehicle (for both ACC vehicles or manual vehicles). For each vehicle, the following errors are defined:

$$\varepsilon_i = x_{i-1} - x_i - D_i \qquad \text{(Range Error)}$$

$$\varepsilon_{vi} = v_{i-1} - v_i \qquad \text{(Range Rate Error)}$$

where D_i denotes the desired range for the ith vehicle.

In this paper we have assumed constant time-headway policy is adopted for all vehicles, that is, the desired ranges are proportional to vehicle speeds. Let $D_i = h_i \cdot v_i$ (h_i is the constant time-headway for the ith vehicle), then the range errors can be rewritten as:

$$\varepsilon_i = x_{i-1} - x_i - h_i \cdot v_i \tag{2}$$

To investigate the string stability of such a system, a propagation transfer function $\overline{G}_{i,k}$ is defined as the transfer function from range error of ith vehicle to the range error of the $i+k$th vehicle.

$$\overline{G}_{i,k} = \frac{\varepsilon_{i+k}}{\varepsilon_i} \tag{3}$$

Substituting (1) and (2) into (3), we have

$$\overline{G}_{i,k} = G_i \cdot G_{i+1} \cdots G_{i+k-1} \cdot \frac{(1 - G_{i+k} - s \cdot h_{i+k} \cdot G_{i+k})}{(1 - G_i - s \cdot h_i \cdot G_i)} \tag{4}$$

Clearly, the propagation transfer function $\overline{G}_{i,k}$ depends on the following algorithms of all the vehicles between the *i*th vehicle and the *i+k*th vehicle. In the derivation we have not specified G_i. Therefore, this general formulation can be used to study the string stability of uniform vehicle strings as well as mixed vehicle strings in the following section.

3. String Stability of Vehicle Strings

3.1. Uniform Vehicle Strings

Consider a uniform vehicle string, that is, all vehicles in the string are identical, i.e. $G_i = G$ and $h_i = h \ \forall \ i$. It is clear that the range error output must be smaller than or equal to the range error input to avoid range errors propagate indefinitely along the string. For this uniform vehicle string, a string-stability definition is widely used [7] and is described as following:

Definition 1 (String Stability of A Uniform String):

A uniform vehicle string is string stable if

$$\|\varepsilon_{i+1}\|_2 \leq \|\varepsilon_i\|_2$$

Remark:

In a uniform vehicle string, the propagation transfer function $\overline{G}_{i,1}$ from the range error (ε_i) of one vehicle to the range error (ε_{i+1}) of its follower can be written as:

$$\frac{\varepsilon_{i+1}}{\varepsilon_i} = \overline{G}_{i,1} = G_i \cdot \frac{(1 - G_{i+1} - s \cdot h_{i+1} \cdot G_{i+1})}{(1 - G_i - s \cdot h_i \cdot G_i)} = G_i = G$$

To achieve string stability, the inequality $\|G\|_\infty \le 1$ needs to be satisfied. Therefore, the string stability of an uniformed vehicle string can be determined by the car-following algorithm *G*.

3.2. Mixed Vehicle Strings

In the previous section, string stability is defined under the assumption of uniform vehicle strings. On real highway, however, a vehicle strings consist of different types of vehicles, including manual and automated, string stable and string unstable vehicles. What is the string-stability property of such a mixed vehicle string? More specifically, if we consider a mixed vehicle string consisted of string-unstable manual vehicles and string-stable semi-automated vehicles, how can we define the string stability of this mixed vehicle string? Clearly, the string-stability definition in **Definition 1** is not enough to answer these questions. In order to investigate this problem, we first define string stability for mixed vehicle strings.

Fig 5 For a mixed vehicle string, the string stability from vehicle to vehicle has become meaningless because no simple expression can represent all vehicles in this mixed vehicle string. For example, if there are three vehicles in a string following a lead vehicle (Fig. 5) and they are all string-stable under **Definition 1** with a constant time-headway *h* = 1 sec,

$$G_1(s) = \frac{0.7s+1}{s^2+1.7s+1}$$
$$G_2(s) = \frac{0.5s+1}{s^2+1.5s+1}$$
$$G_3(s) = \frac{0.7s+1}{s^2+1.7s+1}$$

with

$$\|G_1\|_\infty \le 1, \quad \|G_2\|_\infty \le 1, \quad \|G_3\|_\infty \le 1$$

the two propagation transfer functions are as follows

$$\overline{G}_{1,1} = \frac{\varepsilon_2}{\varepsilon_1} = G_1 \cdot \frac{(1 - G_2 - s \cdot h \cdot G_2)}{(1 - G_1 - s \cdot h \cdot G_1)}$$

$$\overline{G}_{2,1} = \frac{\varepsilon_3}{\varepsilon_2} = G_2 \cdot \frac{(1 - G_3 - s \cdot h \cdot G_3)}{(1 - G_2 - s \cdot h \cdot G_2)}$$

and we found that

$$\|\overline{G}_{1,1}\|_\infty > 1, \quad \|\overline{G}_{2,1}\|_\infty < 1$$

It is obviously that no conclusion about the string stability of this mixed string can be drawn in this numerical example. In the following, we will propose a string-stability definition for mixed vehicle strings.

Fig 6 Consider a mixed vehicle string (S_1) of k vehicles. If this string is repeated to form an infinite string as in Fig. 6, then the propagation transfer function from the first vehicle's rang error (ε_{nk+1}) of one sub-string (S_n) to that ($\varepsilon_{(n+1)k+1}$) of the following sub-string (ε_{n+1}) is as follows:

$$\begin{aligned}\overline{G}_{nk+1,k} &= \frac{\varepsilon_{(n+1)k+1}}{\varepsilon_{nk+1}} \\ &= G_{nk+1} \cdot G_{nk+2} \cdots G_{nk+k} \cdot \frac{(1 - G_{(n+1)k+1} - s \cdot h_{(n+1)k+1} \cdot G_{(n+1)k+1})}{(1 - G_{nk+1} - s \cdot h_{nk+1} \cdot G_{nk+1})}\end{aligned} \tag{5}$$

$n = 0..\infty$

Because $G_{nk+1} = G_1$, $G_{nk+2} = G_2$, $\cdots$, $G_{nk+k} = G_k$ and $h_{nk+1} = h_1$, (5) becomes

$$\overline{G}_{nk+1,k} = G_1 \cdot G_2 \cdots G_k \qquad n = 0..\infty \qquad (6)$$

It is clear that to avoid any range error being amplified unboundedly along this imaginary infinite vehicle string, the magnitude of $\overline{G}_{nk+1,k}$ must be less than or equal to 1. Therefore, the string-stability definition of a mixed vehicle string is stated as follows:

Definition 2 (String Stability of Mixed Vehicle Strings):

A mixed vehicle string of k vehicles is string stable if $\left\|\overline{G}_{nk+1,k}\right\|_\infty \le 1$. That is, $\left\|G_1 \cdot G_2 \cdots G_k\right\|_\infty \le 1$ where G_i, $i = 1 .. k$, represents the car-following algorithms of the ith vehicle.

Remark 1:

If a vehicle string is string stable and all vehicles in this string are identical, then each vehicle must be string stable. It is obvious that **Definition 1** is just a special case of **Definition 2**.

Remark 2:

According to **Definition 2**, the string stability of a mixed vehicle string is not affected by the position of individual vehicle in this string.

Theorem 1:

A mixed vehicle string is string stable if all vehicles in this string is string stable.

Proof:

$$\left\|\overline{G}_{1,k}\right\|_\infty = \left\|G_1 \cdot G_2 \cdots G_k\right\|_\infty \le \left\|G_1\right\|_\infty \cdot \left\|G_2\right\|_\infty \cdots \left\|G_k\right\|_\infty$$

If all the vehicles are string stable, i.e. $\|G_i\|_\infty \le 1$, i = 1..k, we have $\|\overline{G}_{1,k}\|_\infty \le 1$. From definition 2, this mixed vehicle string is string stable.

4. String Stability Margin (SSM)

String stability has become an important design issue in the vehicle longitudinal control. Researches have been done on the proof and analysis of string stability. However, no quantitative measurement of string stability has been provided. As a result, there is no way we can determine if one ACC design is "marginally" string stable? Or if one ACC design is more string stable than the other? In this section, we will define a string-stability margin (SSM) and determine the string stability of ACC designs in the context of SSM. The margin is basically measure of how close an ACC design comes to the marginal string stability, i.e. $\|G\|_\infty = 1$. The operational definition of SSM is stated below

Definition 3 (String-Stability Margin):

Consider a mixed vehicle string consisting of n standard manual vehicles with their car-following algorithm represented by G_{MV} and an ACC controlled vehicle with its car-following algorithm represented by G_{ACC}. Increase n from zero until n_{max} so that the following inequality is not satisfied

$$\|\underbrace{G_{MV} \cdot G_{MV} \cdots G_{MV}}_{n} \cdot G_{ACC}\|_\infty \le 1$$

The n_{max} is the SSM for this ACC controlled vehicle.

Remark 1:

If n_{max} is equal to zero, then this ACC design is marginally string stable. The larger n_{max} is, the more string-stable the ACC design is.

In order to measure SSM, the Pipes human driver model [12] is used as a standard manual vehicle model.

$$M \cdot \dot{v}_i(t) = \lambda \cdot \left[v_{i-1}(t-\Delta) - v_i(t-\Delta) \right]$$

where M is vehicle mass, λ is the sensitivity of the control mechanism, and Δ is the time delay of human driver. From Chandler's paper [10], the average value of λ/M is equal to 0.368 and the average value of Δ is equal to 1.55. The input-output behavior of this human car-following model can be approximated by the following linear transfer function

$$G_{MV} = \frac{v_i}{v_{i-1}} = \frac{0.368 \cdot e^{-1.55s}}{s + 0.368 \cdot e^{-1.55s}} \approx \frac{-0.57s + 0.74}{1.55s^2 + 1.43s + 0.74}$$

In the following, we will examine the SSM for an optimal ACC algorithm design [2], which considers not only the behavior of the controlled vehicle itself, but also all its following vehicles.

Consider an infinite vehicle strings. Assume all the vehicles in this string are identical and are under the same control strategy. Each vehicle can be represented by the following dynamics equation:

$$\dot{x}_k = A_k x_k + B_k u_k \quad k = -\infty..\infty \tag{7}$$

where $x_k = \begin{bmatrix} z_k \\ v_k \end{bmatrix}$, $A_k = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_k = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, z_k is the position of the *k*th vehicle and v_k is the velocity of the *k*th vehicle. When a constant time-headway policy is used, the two major error terms are $z_{k-1} - z_k - h_d v_k$ (range error) and $v_{k-1} - v_k$ (range rate error). Defining these two variables as outputs for each vehicle, we have

$$y_k = \sum_{j=-\infty}^{\infty} G_{k-j} x_j \tag{8}$$

$$G_{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad G_0 = \begin{bmatrix} -1 & -h_d \\ 0 & -1 \end{bmatrix}, \quad G_k = 0, \quad \forall k \neq -1,0$$

If we use simple proportional control, the control law becomes

$$u_k = k_1 \cdot (z_{k-1} - z_k - h_d v_k) + k_2 \cdot (v_{k-1} - v_k) \equiv K \cdot y_k \tag{9}$$

An optimal control framework can then be formulated to minimize the range errors and range rate errors for all the vehicles in the string. A performance index can thus be defined:

$$J = \frac{1}{2} \int_0^{\infty} \sum_{k=-\infty}^{\infty} [q_1 (z_{k-1} - z_k - h_d v_k)^2 + q_2 (v_{k-1} - v_k)^2 + r u_k^2] dt \tag{10}$$

where q_1, q_2, and r are the design penalties on the range error, range-rate error, and control effort respectively.

A bilateral *z* transformation technique [11] is applied to solve this optimization problem. The dynamics equations are transformed to the following equations:

$$\dot{x}(z) = A(z)x(z) + B(z)u(z) \tag{11}$$

$$y(z) = G(z)x(z) \tag{12}$$

$$u(z) = Ky(z) \tag{13}$$

The performance index in Eq. (10) can also be rewritten as

$$\hat{J} = \left\langle \frac{1}{2} tr \int_0^\infty \exp(\mathbf{D}(z^{-1}, K)^T \cdot t)\mathbf{M}(z, K)\exp(\mathbf{D}(z, K) \cdot t) \cdot dt \right\rangle_0 \tag{14}$$

where

$$\mathbf{D}(z, K) = \mathbf{A}(z) + \mathbf{B}(z)K\mathbf{G}(z) \tag{15}$$

$$\mathbf{M}(z, K) = \mathbf{Q}(z) + \mathbf{G}(z^{-1})^T K^T \mathbf{R}(z)K\mathbf{G}(z) \tag{16}$$

The objective of this optimization problem is to find an optimal gain K^* that minimizes the performance index $\hat{J}$. K^* can be solved by the following equation using an iteration method.

$$\begin{aligned} K = -\frac{1}{r}&\left\langle \mathbf{B}(z^{-1})^T \mathbf{P}(z,K)\mathbf{L}(z,K)\mathbf{G}(z^{-1})^T + \mathbf{B}(z)^T \mathbf{P}(z,K)^T \mathbf{L}(z,K)^T \mathbf{G}(z)^T \right\rangle_0 \cdot \\ &\left\langle \mathbf{G}(z)\mathbf{L}(z,K)\mathbf{G}(z^{-1})^T + \mathbf{G}(z^{-1})\mathbf{L}(z,K)^T \mathbf{G}(z)^T \right\rangle_0^{-1} \end{aligned} \tag{17}$$

where the matrices $\mathbf{P}(z, K)$ and $\mathbf{L}(z, K)$ are solved from the following algebraic equations:

$$\mathbf{P}(z, K)\mathbf{D}(z, K) + \mathbf{D}(z^{-1}, K)^T \mathbf{P}(z, K) + \mathbf{M}(z, K) = 0 \tag{18}$$

$$\mathbf{L}(z, K)\mathbf{D}(z^{-1}, K)^T + \mathbf{D}(z, K)\mathbf{L}(z, K) + I_n = 0 \tag{19}$$

More detailed derivation can be found in [2]. An advantage of this optimal ACC design is that if there exists an optimal control gain K^*, the gain is guaranteed to make the controlled ACC vehicle string stable.

Table 1 shows the optimal control gains and their corresponding SSM values under different penalties with the constant time-headway h = 1.4 sec.

Table 1

5. SIMULATION

In this section, the optimal ACC design described in Section 4 are implemented for simulation studies. The desired constant time-headway in the control design is selected to be 1.4 seconds, which is the average value taken from the FOCUS field test data. The optimal control gain K^* =[1.12,1.70] correspond to the penalties $q_1 = 1$, $q_2 = 1$, and $r = 1$. Two different simulation tools are used to perform a series of simulations to demonstrate the effectiveness of ACC systems on traffic smoothness. First, we use MATLAB to simulate a platoon of 20 vehicles. These simulations are based on the assumption that the lateral operations of all vehicles are perfect. That is, disturbances due to lane changing /merging are not considered. Second, the UMACC simulator is used to investigate the effectiveness of ACC systems. In this simulation program, the vehicle longitudinal dynamics is simple, but a complex lane change behavior is included. A two-lane closed-circuit highway is constructed in which autonomous lane changes will occur. The fact that we are simulating individual vehicles enables us to study safety and traffic-flow characteristics more accurately.

5.1 MATLAB Study

We first examine the transient behavior in dense manual traffic where a string of 20 manual vehicles follow a lead vehicle in a single lane without passing. The Pipes model is used to represent the manual vehicles. All vehicles start at a constant velocity of 30 m/sec (67.5 mph) and then the lead vehicle accelerates to 32 m/sec (72 mph) and keeps at 32 m/s for 10 sec and decelerates back to 30 m/sec. The acceleration is 1 m/sec^2 and the jerk is limited to 20 m/s^3. Figure 7 shows that the

Pipes (manual) vehicles exhibit slinky effect. Figure 8 shows that $\|v - v_0\|_2$ is amplified upstream.

Fig 7,8

We then consider mixed manual/ACC vehicle strings. Assuming a mixed string of 25% ACC vehicles, we investigate the identical situation when 20 vehicles (manual/ACC) follow a lead vehicle in a single lane. The lead vehicle is given the same maneuver as in previous case. The ACC vehicles are placed uniformly at position 1, 5, 9, 13, and 17. The manual vehicles amplify the velocity errors as seen earlier. However, the ACC vehicles successfully reduce the slinky effect caused by manual vehicles (Fig. 9,10). As a result, the last vehicle (v20) shows a smaller velocity change (compare with v16).

Fig 9,10

5.2 UMACC Study

In the UMACC simulation study, the movement of vehicles on a 20 km two-lane test track are simulated for one hour. The results for different traffic densities and different ACC penetration rates are shown in Table 2. For each selection of traffic density and penetration rate, the average velocity, RMS value of acceleration, and RMS value of range rate of all vehicles are calculated. It can be seen that at low traffic density (7.5 veh/ln/km), the effect of ACC vehicles on traffic is not obvious. As traffic density increases, the effect of ACC vehicles becomes more and more profound. At high traffic density (20 veh/ln/km), for 40% ACC penetration rate, the average velocity increases by 16%, the RMS of acceleration decreases by 27%, and the RMS of range rate decreases by 37 %. The benefit of such an ACC system is obvious. Higher average velocity means higher traffic throughput, lower RMS value of acceleration means lower fuel consumption and lower air pollution, and lower RMS value of range rate not only means smoother but also safer highway traffic.

Table 2

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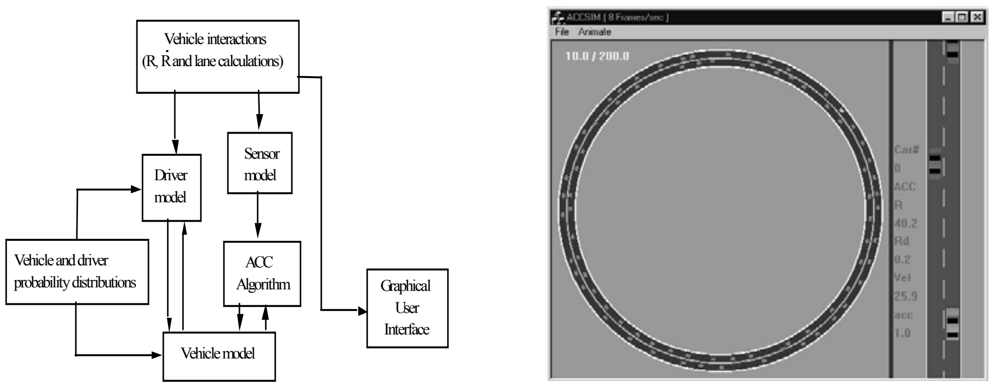

Fig. 1 ACC Simulator

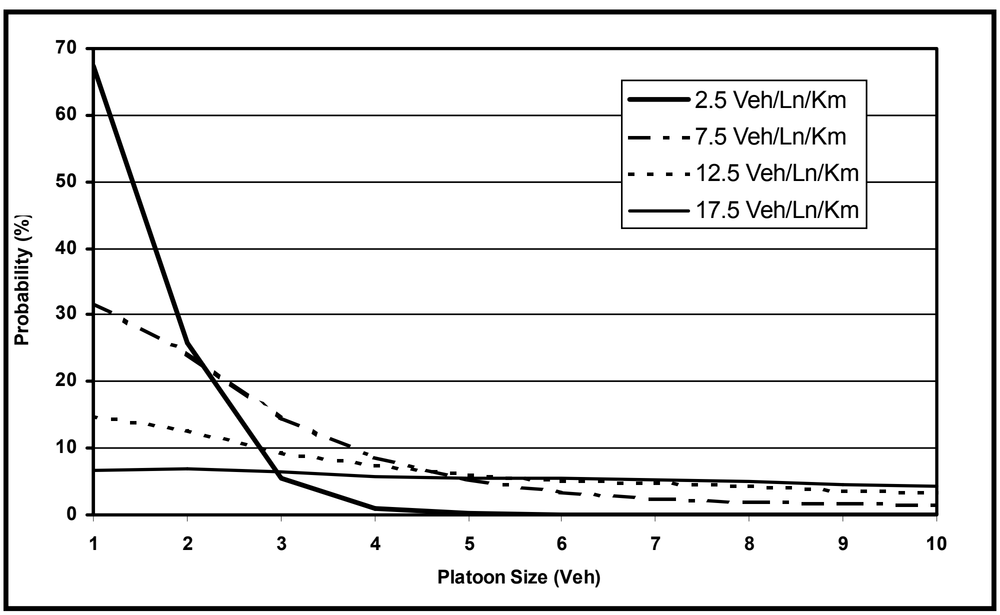

Fig. 2 Probability of a Vehicle in platoons of Different Sizes

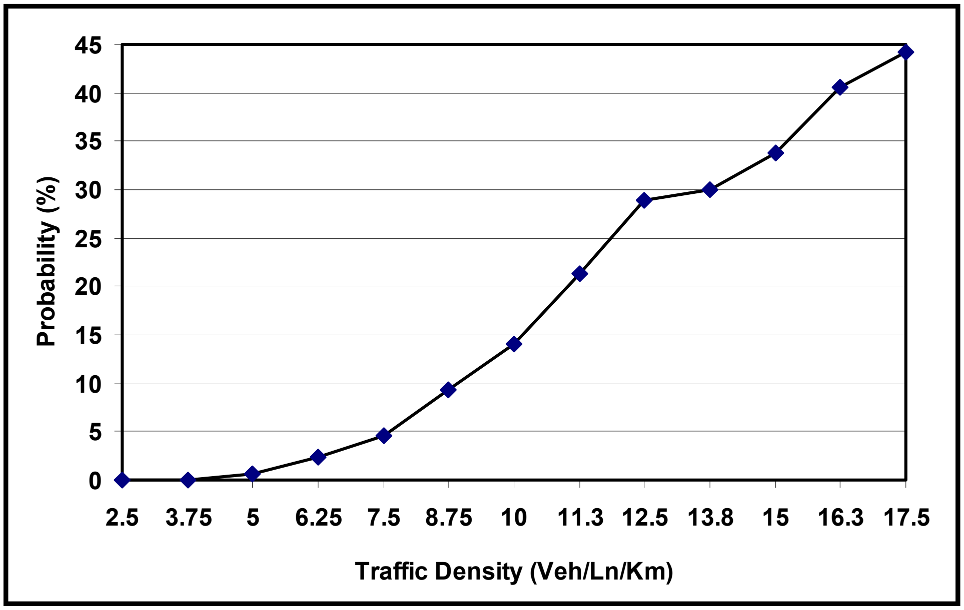

Fig. 3 Probability of a Vehicle in Platoons of Size > 10

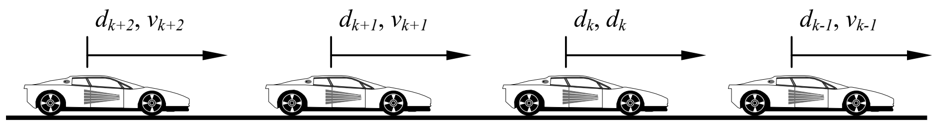

Fig. 4 Vehicle String

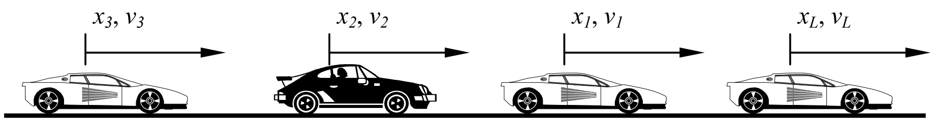

Fig. 5 Mixed Vehicle String

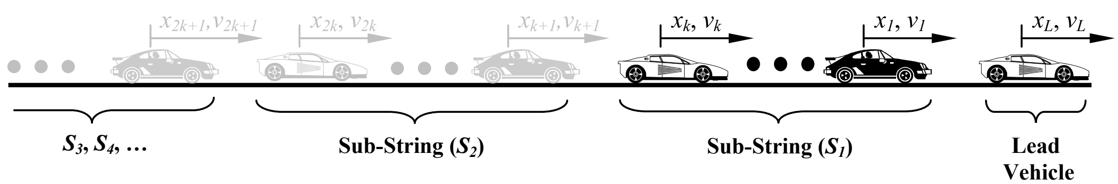

Fig. 6 Infinite Mixed Vehicle String

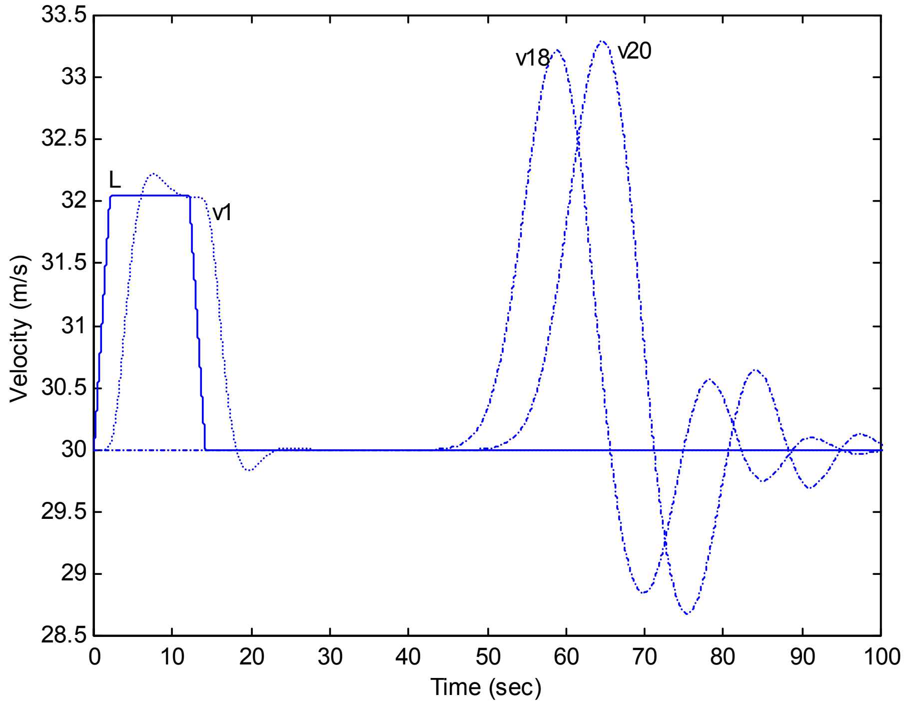

Fig. 7 Velocity profile of the leader and following vehicles

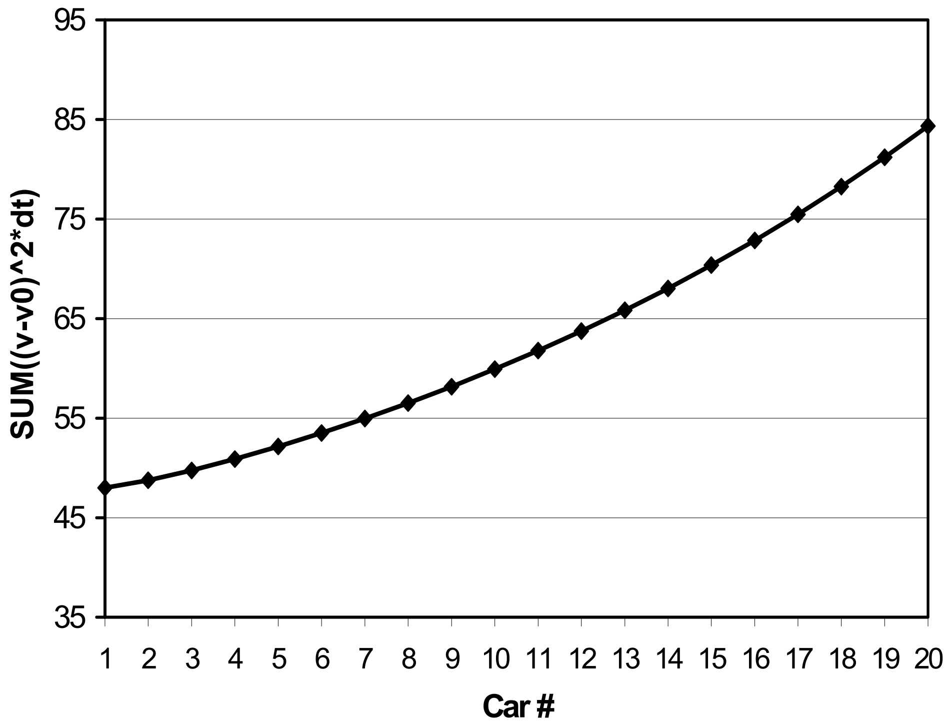

Fig. 8 2-Norm of (v-v_0) of the following vehicles

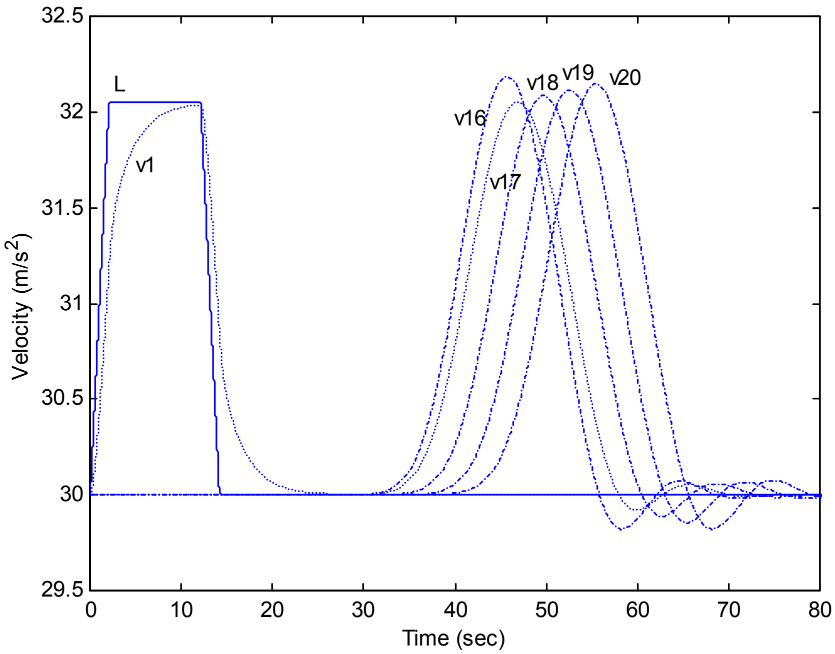

Fig. 9 Velocity Profile of the leader and following vehicles in mixed traffic

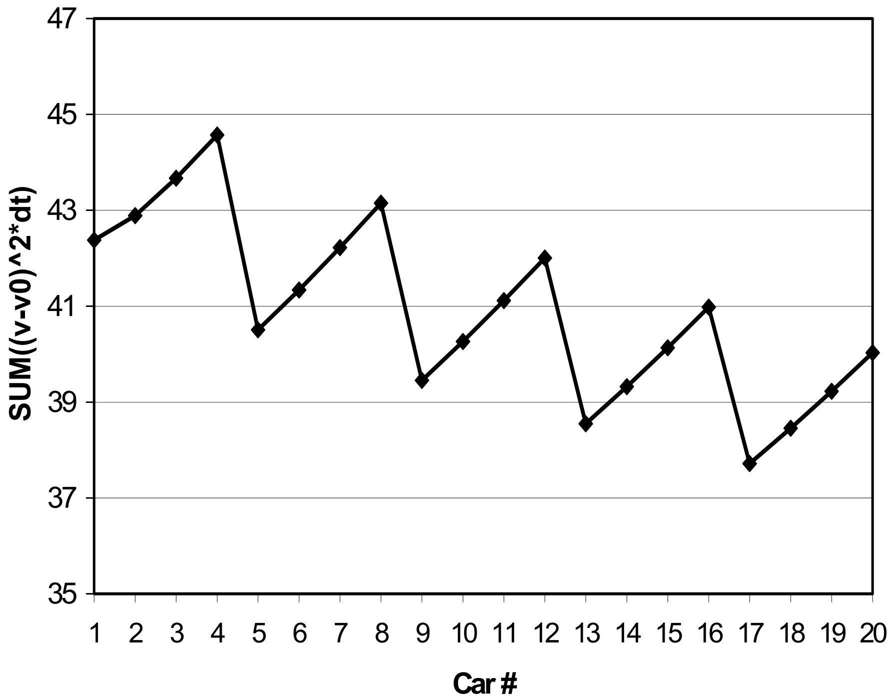

Fig. 10 2-Norm of (v-v_0) of the following vehicles

Table 1 Optimal Control Gains and Corresponding SSM for *h*=1.4 sec

q_1	q_2	r	K^*	SSM
1	1	1	[1.12, 1.70]	4.22
0.1	0.1	1	[0.45, 1.44]	4.80
0.1	10	1	[0.42, 2.15]	4.86
10	0.1	1	[2.20, 2.47]	4.05
10	10	1	[2.10, 2.94]	4.70

Table 2 Effects of the optimal ACC systems under different traffic densities

ACC Penetration Rate		0%	20%	40%	60%	90%
7.5 veh/km/ln	V_avg (m/s)	27.75	27.70	27.71	27.58	27.57
	RMS_a (m/s^2)	0.25	0.27	0.29	0.29	0.32
	RMS_Rd (m/s)	2.26	2.25	2.28	2.17	2.20
16.25 veh/km/ln	V_avg (m/s)	25.85	25.42	25.67	25.78	25.72
	RMS_a (m/s^2)	0.35	0.35	0.30	0.27	0.26
	RMS_Rd (m/s)	1.93	1.90	1.58	1.47	1.35
20 veh/km/ln	V_avg (m/s)	20.19	21.15	23.38	24.64	24.65
	RMS_a (m/s^2)	0.56	0.53	0.41	0.32	0.17
	RMS_Rd (m/s)	3.31	3.08	2.08	1.34	0.70