Disturbance Observer Based Tracking Control

Chia-Shang Liu

Huei Peng
e-mail: hpeng@umich.edu

Department of Mechanical Engineering and
Applied Mechanics,
University of Michigan,
Ann Arbor, MI 48109-2121

A disturbance observer based tracking control algorithm is presented in this paper. The key idea of the proposed method is that the plant nonlinearities and parameter variations can be lumped into a disturbance term. The lumped disturbance signal is estimated based on a plant dynamic observer. A state observer then corrects the disturbance estimation in a two-step design. First, a Lyapunov-based feedback estimation law is used. The estimation is then improved by using a feedforward correction term. The control of a telescopic robot arm is used as an example system for the proposed algorithm. Simulation results comparing the proposed algorithm against a standard adaptive control scheme and a sliding mode control algorithm show that the proposed scheme achieves superior performance, especially when large external disturbances are present.
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1 Introduction

Tracking control for uncertain nonlinear systems with unknown disturbances is a challenging problem. To achieve good tracking under uncertainties, one usually needs to combine several or all of the following three mechanisms in the control design: adaptation, feedforward (plant-inversion), and high-gain, this paper is no exception. The tracking control of nonlinear systems under plant uncertainties and exogenous disturbances is studied in this paper. However, we will focus on the robotic examples for both literature review and numerical simulations.

Many adaptive control schemes for robotic manipulators assume that the structure of the manipulator dynamics is known and/or the unknown parameters influence the system dynamics in an affine manner [1–5]. There are several inherent difficulties associated with these approaches. First of all, the plant dynamic structure may not be known exactly. Second, it was demonstrated [6,7] that some of these designs may lack robustness against uncertainties. Recently, adaptive control algorithms requiring less model information were proposed [8–11]. These algorithms adjust the control gains based on the system performance and thus are commonly referred to as performance-based adaptive control. These algorithms require little knowledge of system structures and parameter values. However, the control signal might become quite large. Plant-inversion based methods (e.g., I/O linearization, backstepping), roughly speaking, focus on the canceling of unwanted nonlinear dynamics. High-gain approaches such as sliding model controls could guarantee stability but, again, sometimes require very large control signals. While in some cases this may be a viable approach, in many other applications it may not be the best solution.

In this paper, a disturbance-estimation based tracking control method is presented. Disturbance observer based control algorithms first appeared in the late 1980s [12]. Since then, they have been applied to many applications [13–15]. Recently, the H_∞ technique has been applied for the design of an optimal disturbance observer [16]. In this paper, we focus on the design for nonlinear systems. The magnitude of the disturbance is estimated based on the state estimation error in a two-step design. The estimated disturbance can then be used to improve the performance of literally any control algorithms. In this paper, a simple computed torque method is selected. The performance of the disturbance-observer-enhanced method is then compared against those of a simple adaptive control and a simple robust control algorithm.

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2 Disturbance Estimation Based Tracking Control Schemes

The nonlinear systems studied in this paper are assumed to have the following form

$$\dot{x}(t)=Ax(t)+\Gamma(u,x)+Bd \tag{1}$$

where $x\in R^n$ denotes the state vector, $A\in R^{n\times n}$ is the known state matrix, $\Gamma(u,x)\in R^n$ is a known nonlinear vector, $d\in R^m$ is the lumped disturbance vector which includes uncertainties of A, Γ and external disturbances. $B\in R^{n\times m}$ is the known disturbance input matrix. Due to the lumped nature of d, B is usually square. An ''observer'' for this system is

$$\dot{\hat{x}}(t)=A\hat{x}(t)+\Gamma(u,x)+B\hat{d}(t)+K(x-\hat{x}) \tag{2}$$

where $K\in R^{n\times n}$ is the observer gain. The error dynamics are then $\dot{e}=A_k e+Be_d$, where $e=\hat{x}-x$, $e_d=\hat{d}(t)-d(t)$, and $A_k=A-K$ is the closed-loop state matrix. Since we have full state feedback, K can be chosen to make A_k Hurwitz. The disturbance estimation laws are then chosen to be

$$\dot{e}_{d_o}=\dot{\hat{d}}_o=-B^TPe \tag{3}$$

$$\hat{d}(t)=\hat{d}_o(t)-K_oe \tag{4}$$

$$K_oA_k+K_oBK_o+B^TP=0 \tag{5}$$

where $\hat{d}_o(t)$ is the uncorrected estimated disturbance, $\hat{d}(t)$ is the corrected estimated disturbance, and K_o is the correction gain. The disturbance estimation schemes shown in Eqs. (3)–(5) consist of two steps: (i) precorrection, obtained by assuming that the disturbances are constant (under which Eq. (3) becomes true), and (ii) estimation correction for time-varying disturbances. The convergence property of the uncorrected estimation scheme (Eq. (3)) is summarized in the following two facts.

Fact 1: If d is constant and A_k is Hurwitz, then the update law $\dot{e}_{d_o}=-B^TPe$ guarantees $\dot{V}<0$ for the Lyapunov function $V=e^TPe+e_{d_o}^Te_{d_o}$.

Proof: see Appendix.

Fact 2: If the update law (Eq. (3)) is applied to a system with constant disturbances, then (i) $e\in\mathcal{L}^\infty\cap\mathcal{L}^2$ and $e_{d_o}\in\mathcal{L}^\infty$, and (ii) $\lim_{t\to\infty} e(t)\to 0$ and $\lim_{t\to\infty} Be_{d_o}(t)\to 0$. Furthermore, if B has full column rank, then we also have (iii) $\lim_{t\to\infty} e_{d_o}(t)\to 0$.

Proof: see Appendix.

It should be noted that matrix B can always be modified to satisfy the column-rank requirement of Fact 2. The main prob-

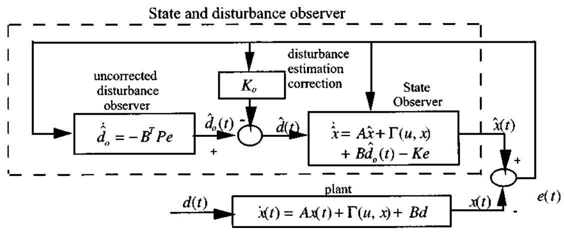

Fig. 1 Schematic diagram of the disturbance estimation algorithm

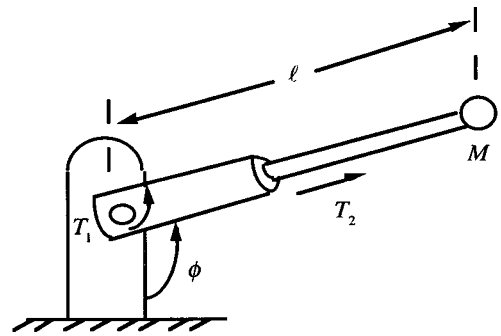

Fig. 2 Telescopic robot arm

lem of Eq. (3) is that when $d(t)$ is time-varying, it cannot be implemented. To relax this restriction, we introduce the following procedure.

Lemma 3: If (i) the assumptions in **Facts 1** and **2** are satisfied, except that $d(t)$ may be time-varying. An improved disturbance estimation is then $\hat{d}(t)=\hat{d}_o(t)-K_o e$, where $e_{d_o}=K_o e$ describes the approximated relationship between e and e_{d_o}, and K_o is solved from Eq. (5).

Proof: see Appendix.

The disturbance estimation procedure is summarized in Fig. 1. It should be noted that Eq. (5) is algebraic, and the existence of a solution is guaranteed. When K_o is low dimensional (like the robotic example to be presented below), it can be solved symbolically. For higher dimensional K_o, a numerical solution might have to be used.

3 Tracking Control Example—Robot Manipulators

3.1 Dynamic Equations of the Robot Manipulator. The dynamics of a robot manipulator in general can be described as

$$M_n(q)\ddot{q}+C_n(q,\dot{q})\dot{q}+g_n(q)=F \tag{6}$$

where $q \in R^n$ is the generalized coordinate, $(\cdot)_n$ denotes nominal functions, $M_n(q) \in R^{n\times n}$ is the inertia matrix, $C_n(q,\dot{q}) \in R^{n\times n}$ includes the Coriolis and centrifugal terms, $g_n(q)$ is the gravity term, and $F \in R^n$ is the control input associated with the generalized coordinate q. Under external disturbances and plant uncertainties, the true plant dynamics are assumed to be $M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)+d(q,\dot{q},t)=F$, where $M(q)=M_n(q)+\Delta M(q)$, $C(q,\dot{q})=C_n(q,\dot{q})+\Delta C(q,\dot{q})$, and $g(q)=g_n(q)+\Delta g(q)$, and $d(q,\dot{q},t) \in R^n$ represents the disturbance input.

3.2 Disturbance Observer Based Tracking Control. If the parameters of the system are exactly known, the computed torque method would generate a control signal $F=M_n(q)(\ddot{q}_d-k_1\dot{e}_d-k_2e_d)+C_n(q,\dot{q})\dot{q}+g_n(q)$, where q_d is the desired trajectory, $e_d=q-q_d$ is displacement error, and k_1 and k_2 are the feedback gains. When there are no uncertainties, the error dynamics are $\ddot{e}_d+k_1\dot{e}_d+k_2e_d=0$, which can be made asymptotically stable by choosing k_1 and k_2. When uncertainties exist, however, the error dynamics become $\ddot{e}_d+k_1\dot{e}_d+k_2e_d+\delta(\ddot{q},\dot{q},q)=0$, where $\delta(\ddot{q},\dot{q},q)=\Delta M(q)\ddot{q}+\Delta C(q,\dot{q})\dot{q}+\Delta g(q)+d(q,\dot{q},t)$. If $\delta(\ddot{q},\dot{q},q)\neq 0$, the system can only be driven to a neighborhood of the desired trajectory. In the following, we will introduce a disturbance observer based tracking control which can be viewed as a "disturbance-observer-enhanced computed torque" scheme.

Lemma 4: For a plant under constant disturbances, if (i) The control input is chosen to be $F=\ddot{q}_d-k_1\dot{e}_d-k_2e_d-\hat{w}_d(q,\dot{q},\ddot{q},t)$, and (ii) the estimation algorithms $\dot{\hat{w}}_{d_o}=-B^TPE_d$ and $\hat{w}_d(q,\dot{q},\ddot{q},t)=\hat{w}_{d_o}(q,\dot{q},\ddot{q},t)-K_oE_d$ are used, where $E_d=[\begin{smallmatrix} e_d \\ \dot{e}_d \end{smallmatrix}]$. Then we can have $\lim_{t\to\infty} e_d(t)\to 0$, $\lim_{t\to\infty} \dot{e}_d(t)\to 0$, and $\lim_{t\to\infty} e_w(t)\to 0$, where $e_w(t)=w_d-\hat{w}_d$.

Lemma 4 is a direct extension of the results presented in Section 2. It should be noted that we do not have to start from the computed torque method. The estimated disturbance can be used to enhance literally any control algorithms.

4 Case Study (Tracking Control of a Telescopic Robot Arm)

4.1 System Description. The schematic diagram of the robot arm is shown in Fig. 2. Two actuators are used to control the angle ϕ and length ℓ. The mass of the extension arm is assumed to be small compared with the payload M. The dynamic equations were found to be $M\ell^2/\alpha_m(\ddot{\phi}+\alpha_1\dot{\phi}+\alpha_2\phi+g/\ell\,\sin(\phi)+d_1)=u_1$ and $M/k_m(\ddot{\ell}+\alpha_3\dot{\ell}+\alpha_4\ell-g\cos(\phi)+d_2)=u_2$, where $\alpha_1=\alpha_f/M\ell^2+2\dot{\ell}/\ell$, $\alpha_2=\alpha_s/M\ell^2$, $\alpha_3=k_f/M$, $\alpha_4=k_s/M-\dot{\phi}^2$, d_1 and d_2 are unknown disturbances, α_s and k_s are the stiffness coefficients, α_f and k_f are the viscous friction coefficients, and u_1 and u_2 are the electrical currents applied to the actuators. The torque in the joint and the force in the arm are assumed to be $T_1=\alpha_m u_1$ and $T_2=k_m u_2$, respectively, where α_m and k_m are unknown constants. In the following subsections, three full-state feedback control algorithms are presented.

4.2 Adaptive Control. A classical adaptive observer [17] is used to estimate the parameters α_m, α_f, α_s, k_m, k_f, and k_s. To implement this algorithm, the dynamics are shown in a different form:

$$\dot{x}=Ax+B\theta^Tf(x,u)+g(x,u) \tag{7}$$

where θ is the unknown vector. The adaptive observer is chosen to be

$$\dot{\hat{x}}=A\hat{x}+B\hat{\theta}^Tf(x,u)+g(x,u)+K(x-\hat{x}) \tag{8}$$

Based on which θ can be estimated from $\dot{\hat{\theta}}=-f^T(x,u)B^TPe$, where $e=\hat{x}-x$, P is a positive definite matrix solved from $A_k^TP+PA_k=-Q$, $Q>0$. More specifically, the update laws are

$$[\dot{\hat{\alpha}}_s \quad \dot{\hat{\alpha}}_f \quad \dot{\hat{\alpha}}_m]=-[-\phi \quad -\dot{\phi} \quad u_1][0 \quad 1]P_\phi\begin{bmatrix}\hat{\phi}-\phi\\ \dot{\hat{\phi}}-\dot{\phi}\end{bmatrix} \tag{9}$$

$$[\dot{\hat{k}}_s \quad \dot{\hat{k}}_f \quad \dot{\hat{k}}_m]=-[-\ell \quad -\dot{\ell} \quad u_2][0 \quad 1]P_\ell\begin{bmatrix}\hat{\ell}-\ell\\ \dot{\hat{\ell}}-\dot{\ell}\end{bmatrix} \tag{10}$$

The control law is then

$$u_1=\frac{M\ell^2}{\hat{\alpha}_m}\left(\ddot{\phi}_d+\hat{\alpha}_1\dot{\phi}+\hat{\alpha}_2\phi+\frac{g\sin(\phi)}{\ell}+k_1\dot{e}_\phi+k_2e_\phi\right) \tag{11}$$

$$u_2=\frac{M}{\hat{k}_m}(\ddot{\ell}_d+\hat{\alpha}_3\dot{\ell}+\hat{\alpha}_4\ell-g\cos(\phi)+k_3\dot{e}_\ell+k_4e_\ell) \tag{12}$$

where $\hat{\alpha}_1=\hat{\alpha}_f/M\ell^2+2(\dot{\ell}/\ell)$, $\hat{\alpha}_2=\hat{\alpha}_s/M\ell^2$, $\hat{\alpha}_3=\hat{k}_f/M$, and $\hat{\alpha}_4=\hat{k}_s/M-\dot{\phi}^2$.

4.3 Sliding Mode Control. The basic idea of sliding mode control is to use switching (saturation) functions with gains larger enough to cover the uncertainties. For the telescopic robot example, the sliding surfaces are chosen to be $S_1=\dot{\phi}-\dot{\phi}_d+\lambda_1(\phi-\phi_d)$ and $S_2=\dot{\ell}-\dot{\ell}_d+\lambda_2(\ell-\ell_d)$, where λ_1 and λ_2 determine the sliding dynamics. The control signals are then obtained from $\dot{S}_1=-\kappa_1\,\mathrm{sat}(S_1/\lambda_1\phi_b)$ and $\dot{S}_2=-\kappa_2\,\mathrm{sat}(S_2/\lambda_2\ell_b)$, where $\mathrm{sat}(\cdot)$ is the saturation function, and ϕ_b and ℓ_b are the widths of the boundary layers. Since this is a standard design process, the detail is omitted.

4.4 Disturbance Observer Based Tracking Control. The robot dynamics are rewritten as $\ddot{\phi}=u_1+w_\phi$ and $\ddot{\ell}=u_2+w_\ell$, where the lumped disturbances are $w_\phi=\ddot{\phi}-M\ell^2/\alpha_m(\ddot{\phi}+\alpha_1\dot{\phi}+\alpha_2\phi+g/\ell\sin(\phi)+d_1)$, and $w_\ell=\ddot{\ell}-M/k_m(\ddot{\ell}+\alpha_3\dot{\ell}+\alpha_4\ell-g\cos(\phi)+d_2)$. From the enhanced computed torque method presented in Section 3.2, the control algorithm is $u_1=\ddot{\phi}_d+k_1\dot{e}_\phi+k_2e_\phi-\hat{w}_\phi$ and $u_2=\ddot{\ell}_d+k_3\dot{e}_\ell+k_4e_\ell-\hat{w}_\ell$. The error dynamics are then $\ddot{e}_\phi-k_1\dot{e}_\phi-k_2e_\phi=e_{w\phi}$ and $\ddot{e}_\ell-k_3\dot{e}_\ell-k_4e_\ell=e_{w\ell}$, where $e_{w\phi}=w_\phi-\hat{w}_\phi$ and $e_{w\ell}=w_\ell-\hat{w}_\ell$. In matrix form, the error dynamics are $\dot{E}_\phi=A_\phi E_\phi+Be_{w\phi}$ and $\dot{E}_\ell=A_\ell E_\ell+Be_{w\ell}$ where $E_\phi=\left[\begin{smallmatrix}e_\phi\\ \dot{e}_\phi\end{smallmatrix}\right]$, $E_\ell=\left[\begin{smallmatrix}e_\ell\\ \dot{e}_\ell\end{smallmatrix}\right]$, $A_\phi=\left[\begin{smallmatrix}0&1\\ k_2&k_1\end{smallmatrix}\right]$, $A_\ell=\left[\begin{smallmatrix}0&1\\ k_4&k_3\end{smallmatrix}\right]$, and $B=\left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$. By applying Fact 1, and if we choose the Lyapunov candidate $V_{(\cdot)}=E_{(\cdot)}^TP_{(\cdot)}E_{(\cdot)}+e_{w(\cdot)}^Te_{w(\cdot)}$, where $P_{(\cdot)}$ satisfies $A_{(\cdot)}^TP_{(\cdot)}+P_{(\cdot)}A_{(\cdot)}=-Q_{(\cdot)}$, and the subscript $(\cdot)$ denotes either ϕ or ℓ. Then the adaptive law $\dot{e}_{w(\cdot)}=-B^TP_{(\cdot)}E_{(\cdot)}$ guarantees $\dot{V}_{(\cdot)}=-E_{(\cdot)}^TP_{(\cdot)}E_{(\cdot)}<0$. Since B has full column rank, from Fact 2 we have $\lim_{t\to\infty}E_{(\cdot)}(t)\to 0$ and $\lim_{t\to\infty}e_{w(\cdot)}\to 0$.

5 Numerical Simulation Results

The simulation results of the three control algorithms shown in the previous section are presented in this section. The nominal plant parameters are assumed to be $M=1.5$, $\overline{\alpha}_m=\overline{k}_m=1.0$, $\overline{\alpha}_s=\overline{k}_s=0.65$, and $\overline{\alpha}_f=\overline{k}_f=0.65$. The true plant parameters are $M=1.5$, $\alpha_m=k_m=0.35$, $\alpha_s=k_s=0.85$, and $\alpha_f=k_f=0.45$. The desired trajectories to be followed are $\phi_d=\pi/2\sin(2\pi t)+\pi$ and $\ell_d=0.2\sin(2\pi t)+1$. In addition to parameter mismatch, external disturbances are assumed to exist, and are $d_1=25\sin(2\pi t)+35\sin(20\pi t)$ and $d_2=25\sin(2\pi t)+15\sin(20\pi t)$, respectively.

Adaptive Control. The observer gains used are $K=\left[\begin{smallmatrix}12&7\\ 12&7\end{smallmatrix}\right]$, $P_\phi=0.1$, and $P_\ell=0.1$, and the control gains are $k_1=28$, $k_2=400$, $k_3=28$, $k_4=400$. The adaptive observer fails to drive the system to the desired trajectory (Fig. 3). This is mainly due to the fact that the parameter estimation does not work properly under external disturbances.

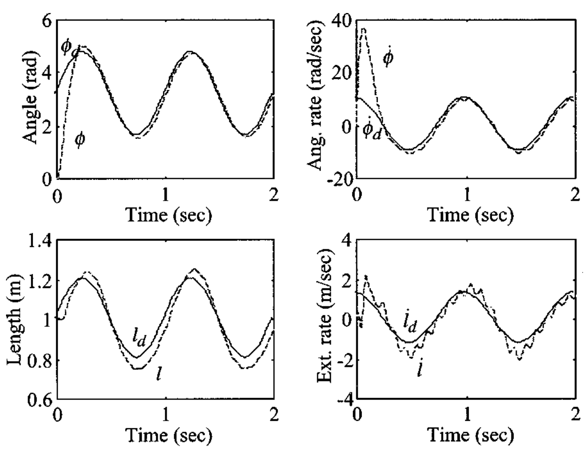

Fig. 3 Simulation results for the adaptive control

Sliding Mode Control. The control gains for the sliding mode control algorithm are $\lambda_1=15$, $\lambda_2=8$, $\phi_b=0.05$, $\ell_b=0.02$, and $\kappa_1=25$, $\kappa_2=25$. Simulation results are shown in Fig. 4. The

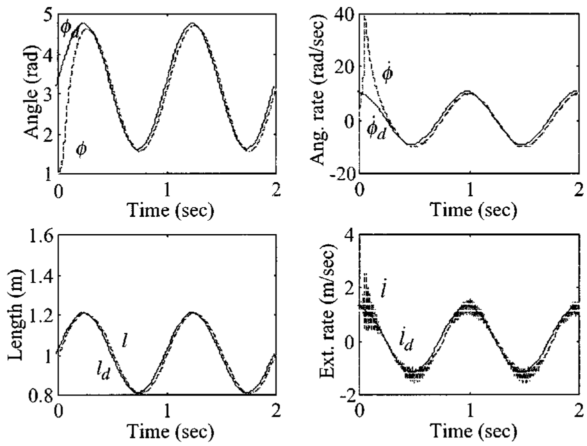

Fig. 4 Simulation results for the sliding mode control

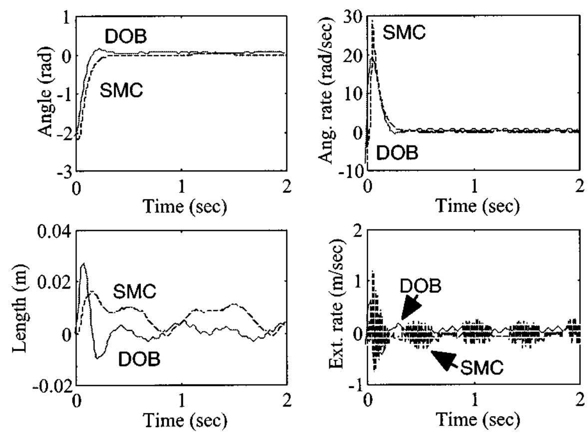

Fig. 5 Tracking errors for disturbance observer based (DOB) and sliding mode control (SMC)

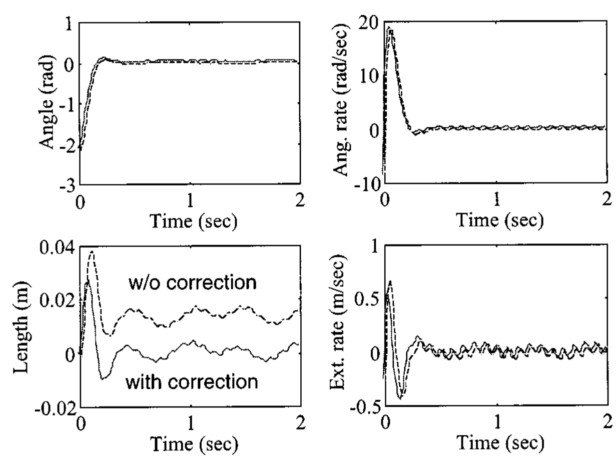

Fig. 6 Tracking error plots for disturbance observer based (DOB) control

tracking performance is adequate, except the arm extension rate. However, the control signal chatters excessively due to the high feedback gains.

Disturbance Observer Based Tracking Control. The observer gains used are $K_o = [47.97\ 7.54]$ and $P = \left[\begin{smallmatrix} 37.92 & 0.21 \\ 0.21 & 0.73 \end{smallmatrix}\right]$, and the control gains are $k_1 = 28$, $k_2 = 400$, $k_3 = 28$, $k_4 = 400$. Since the adaptive control approach fails to work satisfactorily under external disturbances, we will focus on the comparison between sliding mode control (SMC) and the proposed (DOB) algorithm (Fig. 5). The DOB control achieves superior tracking. From the arm extension rate plot, the control torque oscillates much less than that of the SMC.

Finally, Fig. 6 shows the results comparing the DOB controller with and without the estimation correction. It can be seen that when the correction is not used, steady-state error exists in the length tracking. A major benefit of the correction is thus to eliminate the steady-state error caused by external disturbances.

6 Conclusions

A disturbance observer based tracking control scheme for nonlinear systems was proposed. The plant uncertainty, unmodeled dynamics, and external disturbances are lumped into a disturbance term, which is estimated based on the state estimation error in a two-step design. Based on the proposed disturbance estimation scheme, a tracking controller is then constructed which is asymptotically stabilizing in the sense of Lyapunov. The disturbance estimation based tracking control is compared with a classical adaptive controller and a sliding mode controller. A telescopic robot manipulator is used as an application example. The proposed control algorithm was found to generate superior tracking performance and smoother control action. This superior performance is due to the fact that all the system uncertainties are compensated without requiring large feedback gains.

Appendix

Proof of Fact 1:

Choose the Lyapunov candidate $V = e^T P e + e_{d_o}^T e_{d_o}$. The derivative of this function along the trajectory of the error dynamics is then

$$\dot{V} = -e^T Q e + 2e^T P B e_{d_o} + 2\dot{e}_{d_o}^T e_{d_o} \tag{A1}$$

If the adaptive law

$$\dot{e}_{d_o} = -B^T P e \tag{A2}$$

is applied, Eq. $(A1)$ becomes $\dot{V} = -e^T Q e < 0$.

Proof of Fact 2:

(i) Since $V(0) = e^T(0) P e(0) + e_{d_o}^T(0) e_{d_o}(0) \in \mathcal{L}^\infty$ and $\dot{V} = -e^T Q e < 0$, it is obvious $e \in \mathcal{L}^\infty$ and $e_{d_o} \in \mathcal{L}^\infty$. Also, since $\int_0^t \dot{V}\, dt = V(t) - V(0) = -\int_0^t e^T Q e\, dt < \infty$, $e \in \mathcal{L}^2$.

(ii) Since $e \in \mathcal{L}^\infty$ and $e_{d_o} \in \mathcal{L}^\infty$, we have $\dot{e} \in \mathcal{L}^\infty$. From the facts $\dot{e} \in \mathcal{L}^\infty$ and $e \in \mathcal{L}^\infty \cap \mathcal{L}^2$, according to Barbalat's lemma [18], we have $\lim_{t\to\infty} e(t) \to 0$. From $\ddot{e} = A_k \dot{e} + B\dot{e}_{d_o}$ and $(A2)$, we have

$$\ddot{e} = A_k \dot{e} - BB^T P e \tag{A3}$$

Since $e \in \mathcal{L}^\infty$ and $\dot{e} \in \mathcal{L}^\infty$, from $(A3)$, we have $\ddot{e} \in \mathcal{L}^\infty$. Because $\lim_{t\to\infty} \int_0^t \dot{e}(t) dt = -e(0) < \infty$, and $\dot{e}$ is uniformly continuous ($\ddot{e} \in \mathcal{L}^\infty$), from Barbalat's lemma, $\lim_{t\to\infty} \dot{e}(t) \to 0$. Since $\lim_{t\to\infty} \dot{e}(t) \to 0$ and $\lim_{t\to\infty} e(t) \to 0$, from, $\dot{e} = A_k e + B e_{d_o}$, we have $\lim_{t\to\infty} B e_{d_o}(t) \to 0$. If B has full column rank, we can conclude that $\lim_{t\to\infty} e_{d_o}(t) \to 0$.

Proof of Lemma 3:

From $\dot{e} = A_k e + B e_{d_o}$, $e(t)$ is generated from a linear dynamics excited by $e_{d_o}(t)$. Intuitively, we can assume that

$$e_{d_o}(t) = K_o(t) e(t) \tag{A4}$$

is a good low-frequency approximation of the relationship between e_{d_o} and e, where $K_o(t)$ is potentially time varying. In order to satisfy the condition that $\dot{V} < 0$, Eq. $(A4)$ must be consistent with Eq. $(A2)$. Taking the derivative of Eq. $(A4)$

$$\begin{aligned} \dot{e}_{d_o}(t) &= \dot{K}_o(t) e(t) + K_o(t) \dot{e} \\ &= \dot{K}_o(t) e(t) + K_o(t)(A_k e(t) + B e_{d_o}(t)) \end{aligned} \tag{A5}$$

Substituting $\dot{e}_{d_o} = -B^T P e$ and $e_{d_o}(t) = K_o(t) e(t)$ into Eq. $(A5)$, we have

$$\dot{K}_o(t) = -(K_o(t) A_k + K_o(t) B K_o(t) + B^T P) \tag{A6}$$

Eq. $(A6)$ can be integrated in real-time. Or, to simplify the implementation, the steady-state solution can be used:

$$K_o A_k + K_o B K_o + B^T P = 0 \tag{A7}$$

From Eq. $(A4)$, the disturbance estimation correction is then $\hat{d}(t) = \hat{d}_o(t) - K_o e$.

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