Prius^{+} and Volt^{-}: Configuration Analysis of Power-Split Hybrid Vehicles with a Single Planetary Gear

Xiaowu Zhang, Chiao-Ting Li, Dongsuk Kum, Huei Peng

***Abstract*—The majority of the hybrid electric vehicles (HEVs) available on the market are power-split hybrid vehicles with a single planetary gear (PG), including the popular Toyota Prius and Chevy Volt. Although both vehicles use a single PG, they have different configurations with different number of operating modes. The Prius has no clutch and has a single operating mode, whereas the Chevy Volt uses three clutches and has four modes. The goal of this paper is to present a thorough analysis on all possible configurations of power-split hybrid powertrain using a single planetary gear. The analysis includes: 1) search for all possible ways to connect powertrain elements to the PG, 2) identify all potential locations for clutch installations around the PG and examine the feasibility of additional operating modes introduced by the clutch installation; and, 3) optimize the fuel economy for performance comparison. The proposed analysis shows that a single PG can produce 12 different configurations, and each of which can have four feasible operating modes by adding three clutches to the PG. In case studies, we focus on the two configurations that are used in the Prius and Volt in order to find the impact of the additional (or removal of) clutches and modes on their fuel economy performance. Our results show that adding one clutch to the Prius transmission (which is named 'Prius+') can significantly improve the fuel economy in urban driving, while removing two clutch from the Volt transmission ('Volt-') will not significantly affect the fuel economy in both urban and highway driving. This multi-mode configuration analysis can be used to systematically design future power-split HEVs.**

***Index Terms*—power split hybrid vehicles, configuration analysis, dynamic programming, optimal design**

I. Introduction

The hybrid electric vehicle (HEV) market has been dominated by the power-split configuration. The configuration here refers to the how the powertrain elements (the engine, electric machines and the output shaft to vehicle wheels) are connected to the transmission, which usually consists of one or multiple planetary gear sets on HEVs. In 2010, roughly 90% of the HEVs sold were power-split hybrid vehicles using a single planetary gear (PG) [1]; two famous examples are the Toyota Prius and Chevy Volt. The popularity of the power-split hybrids can be attributed to their capability to take advantages of both series and parallel configurations when their power management algorithms are properly designed [2, 3]. Nevertheless, selecting a power-split configuration is complicated due to the large candidate pool and the coupled kinematics among powertrain elements. For example, using a single PG as the split device can produce 12 configurations and each configuration can have up to eight modes if six clutches are added, whereas two PGs can produce 1,152 configurations and each configuration will have at least two modes when two clutches are added [4]. To further complicate the problem, the configuration selection is very much dependent on drive-cycles and vehicle weights. For example, the 2-PG split configurations that offer the best fuel economy in launching a combat vehicle [4] and in city-driving for a delivery truck [5] are different even though similar methodologies are used to screen the same configuration candidate pool. Another complication of the configuration selection is the legal barrier posed by existing patents, such as those owned by Toyota and GM using one or two PGs [6-12], and even three PGs [13]. Companies come to the game late are left with relatively limited options, for example, Ford and Nissan have licensed the 1-mode Toyota technology, and Chrysler and BMW have licensed the GM dual-mode technology [4].

Prior studies have indicated that there exist a large number of possible configurations for 2-PG power-split hybrid vehicles [4, 5], and systematic screening and analysis on all possible power-split configurations will identify the best configuration for the particular vehicle weight class and driving conditions (or runner-up configurations to avoid patent infringement if necessary). The fact that the best configurations for hybrid electric vehicles found in [4] and for hybrid hydraulic vehicles found in [5] are different is a proof of this concept. Systematic screening and analysis of the power split configurations requires two enabling techniques: the generic model representation that can exhaustively search all possible configurations and generate corresponding dynamic models; and, the optimal energy management that can reveal the best execution of each configuration. The first ensures that the search is thorough and the second ensures that all configurations are assessed based on their best performance, so the winning configuration identified

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is truly the optimum.

Mathematical models that describe the kinematic constraints of planetary gear-sets have been reported in the literature [14]. The lever diagram provides a concise representation and has been widely used [15]. Applying permutation on the lever diagram can thoroughly search all power-split configuration candidates, and a screening methodology for the 2-PG split configuration has been developed and used to design HEVs [4] and hydraulic hybrid vehicles [5]. In concept, this screening methodology can be applied to any power-split hybrid vehicle with minor modifications in the kinematics screening rules. A lumped-parameter approach has been proposed as an alternative to the permutation for the 2-PG case to skip over repeated configurations and reduce computational loads [16], but it does not seem to be applicable generally, such as to the single-PG case.

For the optimal energy management of HEVs, various schemes have been reported which include load leveling [17, 18], the Equivalent Consumption Minimization Strategy (ECMS) [19-21], the Pontryagin's minimum principle [22-24], and the dynamic programming (DP) [25, 26]. DP, however, is the only approach that guarantees the global optimality over the problem horizon, but it has a major shortcoming of being very computationally intensive. A comprehensive comparison of HEV energy management approaches can be found in [27]. For HEV studies, DP is generally useful in providing the benchmark best-execution performance, and thus is useful in the design stage (not for real-time control implementation).

Currently, the Toyota Synergy Drive implemented in most Toyota hybrid vehicles, including the Prius, uses an input-split configuration with one PG. The Chevy Volt, distinguishing itself by using a large battery to have a longer electric driving range, uses an output-split configuration with one PG as well. A major difference between these two designs lies in the fact that the Prius does not use any clutch whereas the Volt has three clutches and four operating modes. The different choices of clutch engagements and operating modes inspire us to ask the following questions: how many different configurations for hybrid vehicles can be designed using a single PG? How many clutches can be added, and how many operating modes can they have? What are the benefits of the added clutches and operating modes? In order to fully answer these questions, we need to thoroughly search all configurations, analyze all possible operating modes, and find out their best-execution performances in fuel economy, so that all designs can be compared sensibly.

The remainder of this paper is organized as follows: Section II describes the analysis of the single planetary gear and all possible configurations of hybrid electric vehicles; Section III discusses the addition of clutches on the planetary gear set and the associated operating modes; Section IV presents the dynamic models for all operating modes; Section V states the DP formulation for optimal power management; Section VI presents case studies of the Prius and Volt; and finally, Section VII provides concluding remarks.

II. Single-PG Analysis

Hybrid vehicle configurations refer to the different ways of connecting powertrain elements, including an engine, two electric machines, and an output shaft to the vehicle wheels, to a power-split device, which is a single PG in this paper. In this section, the basic mechanisms of a single PG and possible configurations are described for further developments in clutch placements.

Fig. 1 shows a PG and its equivalent lever diagram [15]. The three nodes on the lever diagram represent the ring gear, carrier, and the sun gear of the PG, and each node can be connected to one or more powertrain elements. The speeds of the ring gear (ω_r), sun gear (ω_s), and carrier (ω_c) must satisfy the kinematic constraint in Eq. (1) [15]:

$$\omega_s S + \omega_r R = \omega_c \left(R + S \right) \qquad (1)$$

where S and R are the radii of the sun gear and ring gear. This kinematic constraint can be visualized by the lever diagram shown on the right in Fig. 1, where the lever lengths represent the relative ratio of S and R, and the three vectors represent the direction and magnitude of speeds. The three speed vectors all touching the straight dash line shows that the kinematic constraint, Eq. (1), is satisfied.

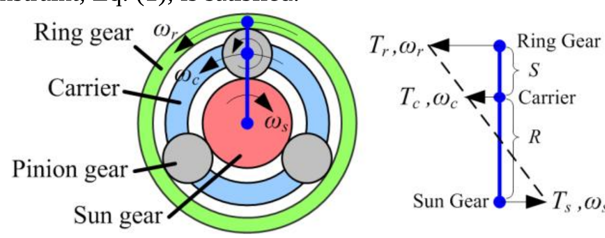


Fig. 1. Planetary gear and its lever diagram [28].

A mechanically feasible configuration must have all of the three PG nodes be connected to at least one powertrain element. No node should be left "hanging freely"—because free node cannot provide any reaction torque. The permutation starts with assigning the engine, output shaft and one electric machine to the three PG nodes, which gives us six possibilities ($P_3^3 = 6$). The second electric machine is then randomly assigned to one of the three nodes. However, having the two electric machines on the same node makes little sense, so we really have only two choices, either on the node with the engine or the node with the output shaft. When the second electric machine is collocated with the output shaft, it is called an input-split configuration; when it is collocated with the engine, it is called an output-split configuration [2]. Therefore, there are a total of 12 possible configurations ($P_3^3 \times 2 = 12$) for HEVs with one PG; six are input-split type (one of which is used for Prius) and six are output-split type (one of which is used for Volt).

III. Clutches and Operating Modes

The use of clutches can introduce various functionalities for single-PG HEV transmissions. For instance, a clutch can disengage the engine from the transmission so that an HEV can operate in the pure electric drive mode. It can also ground a node to use the PG as a simple step gear, which could be useful during a vehicle-launch. Finally, they can disconnect the output shaft so that the engine can charge the battery while the vehicle is stationary. Nevertheless, this last possible mode is dismissed in this paper as it goes against the desire to displace fossil fuel with electricity, and we believe such desperate charging can be avoided by intelligent power management.

The clutch placements around the PG determine the number of operating modes and its characteristics. In order to find all feasible multi-mode single PG configurations, we start the permutation of clutch locations without any constraints, and then eliminate those that are not feasible.

Let us start with an input-split configuration (Fig. 2) as an example. Note that this is only one of the six input-split configurations, and up to six clutches can be added on this particular configuration. In fact, a seventh clutch could have been added to disconnect the output shaft from the PG, but this mode is not considered in this paper because this mode leads to stationary electricity generation. The six clutches in Fig. 2 can be grouped into three pairs, and the two clutches on the same node need to accommodate each other, meaning when one clutch is open, the other must be closed, and vice versa. Therefore, there are eight possible modes (2^3=8). The states of clutches in these eight modes are summarized in Table I and characteristics of each mode are detailed below.

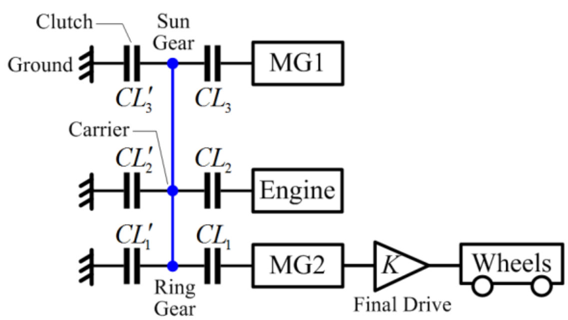


Fig. 2. All possible clutch locations on an input-split configuration.

TABLE I
CLUTCH STATES & OPERATING MODES OF AN INPUT-SPLIT CONFIGURATION

Mode	CL_1	CL_2	CL_3
1 (EV_1)	0	0	1
2 (EV_2)	1	0	1
3 (Series)	0	1	1
4 (Split)	1	1	1
5 (= EV_1)	0	0	0
6 (Infeasible)	1	0	0
7 (= EV_1)	0	1	0
8 (Not EVT*)	1	1	0

Description: "1" means that the clutch is closed; "0" means that the clutch is open. *EVT: Electrically Variable Transmission.

1) Mode 1 is a pure electric mode (EV1). In this mode, clutch CL1 is open, so the vehicle is driven by only the second electric machine (MG2).
2) Mode 2 is also a pure electric mode (EV2). The engine is disconnected and the carrier gear is grounded. The vehicle is driven by both MG1 and MG2.
3) Mode 3 is a series mode (Series). Both the engine and MG1 are connected to the PG to charge the battery, but the vehicle is only driven by MG2 mechanically because clutch CL1 is open. The vehicle is running as a series hybrid vehicle.
4) Mode 4 is a power-split mode (Split). The engine, MG1, and MG2 are all connected to the PG. The vehicle is running as a split hybrid vehicle.
5) Mode 5 is equivalent to Mode 1. Both the engine and MG1 are disconnected, and the vehicle is driven by only MG2.
6) Mode 6 is infeasible. MG2 is locked by the grounded sun ear and carrier, and the output shaft cannot rotate.
7) Mode7 is equivalent to Mode 1. The engine and MG1 are disconnected, and the vehicle is driven by only MG2.
8) Mode8 is impractical. MG1 is disconnected. However, the engine is connected with MG2 at a fixed ratio, in which the PG cannot function as an EVT.

The analysis shows that only four of the eight modes (Modes 1, 2, 3 and 4, see Fig. 3) are useful, and only three clutches (CL_1, CL_1' and CL_2') are needed to realize these four modes. Notice that CL_2 is not necessary because our further analysis shows that *grounding* and *disconnecting* the engine are equivalent—both scenarios disable the engine and use the ground to provide the reactive torque. The split mode, Mode 4 shown in Fig. 3-(d), is the only operating mode used on the Prius powertrain.

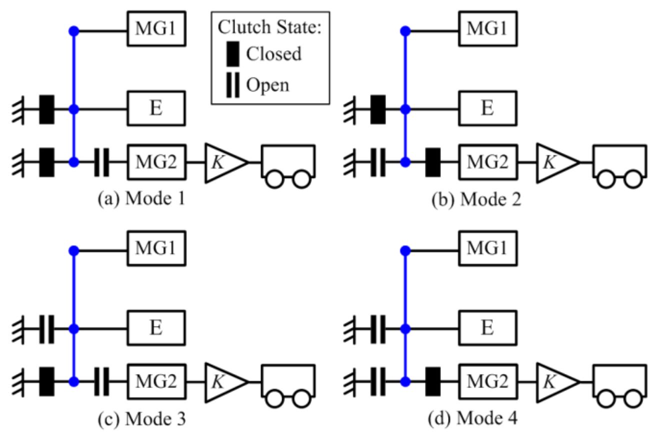


Fig. 3. The four useful operating modes of the input-split configuration.

A similar approach can be applied to all the six output-split configurations, and Fig. 4 shows one example, to which up to five clutches can be added. These clutches can be grouped into two and a half pairs. Again, only four out of the eight possible modes (Modes 1, 2, 3 and 4) are useful (see Table II and Fig. 5), and only three clutches (CL_1, CL_1' and CL_2) are needed to realize these four modes. Note that the Chevy Volt uses exactly this three-clutch arrangement and has all the four modes shown in Fig. 5.

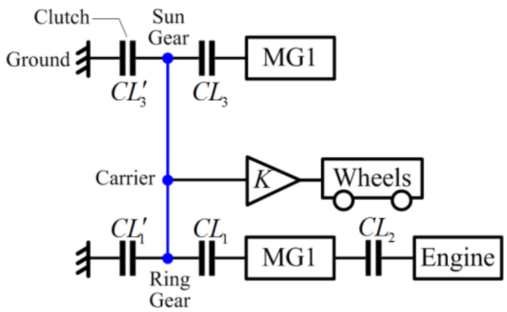


Fig. 4. Five clutches enable eight modes on an output-split configuration.

TABLE II
CLUTCH STATES & OPERATING MODES OF AN OUTPUT-SPLIT CONFIGURATION

Mode	CL_1	CL_2	CL_3
1 (EV1)	0	0	1
2 (EV2)	1	0	1
3 (Series)	0	1	1
4 (Split)	1	1	1
5 (Infeasible)	0	0	0
6 (≈ EV1)	1	0	0
7 (Infeasible)	0	1	0
8 (Not EVT)	1	1	0

Description: "1" means that the clutch is closed; "0" means that the clutch is open.

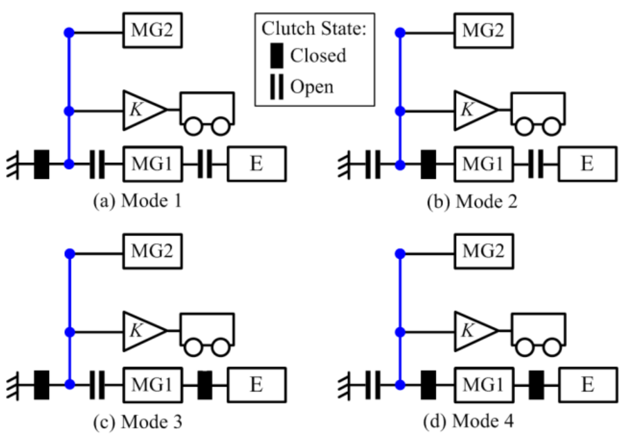


Fig. 5. The four useful operating modes of output-split configuration.

In addition, when adding clutches to the PG, the places where powertrain elements are located respect to the clutches (whether the powertrain element is on the right or left side of the clutch) can also vary. As a consequence, there are other possible topologies for input-split and output-split configurations, shown in Fig 6 and Fig. 7. These alternative topologies may require similar numbers of clutches. However, further analysis shows that these alternatives do not provide any additional functionality compared with the configurations shown in Fig. 2 and Fig. 4. These redundant topologies are shown just for completeness of the paper.

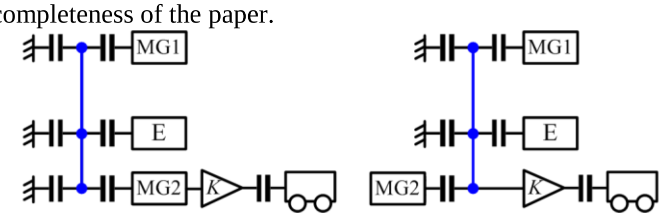


Fig 6. Two other possible topologies for input-split configurations.

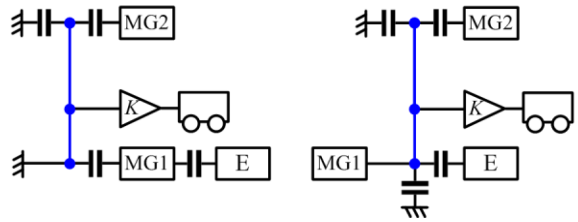


Fig. 7. Two other possible topologies for output-split configurations.

Despite of the fact that the above figures show analysis of clutches and operating modes on only one input-split and one output-split configuration, the analysis applies to all the 12 configurations. In other words, each configuration can all have four operating modes by adding three clutches.

IV. DYNAMIC MODELS

We adopt the generic state-space representation for dynamics of single-PG HEVs in [28] to derive governing equations for all powertrain elements. Equations (2)-(5) describe dynamics of the four operating modes of the input-split configuration shown in Fig. 3. Note that Mode 3 and 4 use four equations to describe the powertrain dynamics, while Mode 1 and 2 require only one and three equations respectively because some powertrain elements are disconnected from the PG.

Mode 1 (EV_1):

$$\left(\frac{mr^2}{K^2}+I_{MG2}\right)\dot{\omega}_{out}=T_{MG2}-T_{load} \tag{2}$$

Mode 2 (EV_2):

$$\begin{bmatrix} \frac{mr^2}{K^2}+I_{MG2} & 0 & -R \\ 0 & I_{MG1} & -S \\ -R & -S & 0 \end{bmatrix}\begin{bmatrix} \dot{\omega}_{out} \\ \dot{\omega}_{MG1} \\ F \end{bmatrix}=\begin{bmatrix} T_{MG2}-T_{load} \\ T_{MG1} \\ 0 \end{bmatrix} \tag{3}$$

Mode 3 (Series):

$$\begin{cases} \left(\frac{mr^2}{K^2}+I_{MG2}\right)\dot{\omega}_{out}=T_{MG2}-T_{load} \\ \begin{bmatrix} I_e & 0 & R+S \\ 0 & I_{MG1} & -S \\ R+S & -S & 0 \end{bmatrix}\begin{bmatrix} \dot{\omega}_e \\ \dot{\omega}_{MG1} \\ F \end{bmatrix}=\begin{bmatrix} T_e \\ T_{MG1} \\ 0 \end{bmatrix} \end{cases} \tag{4}$$

Mode 4 (Power Split):

$$\begin{bmatrix} I_e & 0 & 0 & R+S \\ 0 & \frac{mr^2}{K^2}+I_{MG2} & 0 & -R \\ 0 & 0 & I_{MG1} & -S \\ R+S & -R & -S & 0 \end{bmatrix}\begin{bmatrix} \dot{\omega}_e \\ \dot{\omega}_{out} \\ \dot{\omega}_{MG1} \\ F \end{bmatrix}=\begin{bmatrix} T_e \\ T_{MG2}-T_{load} \\ T_{MG1} \\ 0 \end{bmatrix} \tag{5}$$

where m is the vehicle mass, r is the wheel radius, K is the final drive ratio. I_e, I_{MG1}, I_{MG2} and T_e, T_{MG1}, T_{MG2} are inertia and torques of the engine, first electric machine and second electric machine. T_{load} is the load imposed by the rolling resistance and aerodynamic drag during driving and defined at the transmission output shaft. F is the internal force acting between gears on the PG. ω_e, ω_{MG1} and ω_{out} are speeds of the engine, first electric machine and output shaft. Note that, in this particular configuration, the second electric machine is connected to the output shaft, so its torque acts on the same node at which the output shaft is located, and no additional equation is required to describe its dynamics.

Similarly, the dynamic model for the output-split configuration shown in Fig. 4 can be derived, except that now the first electric machine is connected to the engine, so T_{MG1} will act on the node connected to the engine. Equations (6)-(9) are for the four operating modes of the Volt shown in Fig. 5.

Mode1 (EV_1):

$$\begin{bmatrix} \frac{mr^2}{K^2} & 0 & R+S \\ 0 & I_{MG2} & -S \\ R+S & -S & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}_{out} \\ \dot{\omega}_{MG2} \\ F \end{bmatrix} = \begin{bmatrix} -T_{load} \\ T_{MG2} \\ 0 \end{bmatrix} \tag{6}$$

Mode2 (EV_2):

$$\begin{bmatrix} I_{MG1} & 0 & 0 & -R \\ 0 & \frac{mr^2}{K^2} & 0 & R+S \\ 0 & 0 & I_{MG2} & -S \\ R+S & -R & -S & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}_{MG1} \\ \dot{\omega}_{out} \\ \dot{\omega}_{MG2} \\ F \end{bmatrix} = \begin{bmatrix} T_{MG1} \\ -T_{load} \\ T_{MG2} \\ 0 \end{bmatrix} \tag{7}$$

Mode3 (Series):

$$\begin{cases} (I_e + I_{MG1})\dot{\omega}_{MG1} = T_{MG1} + T_e \\ \begin{bmatrix} \frac{mr^2}{K^2} & 0 & R+S \\ 0 & I_{MG2} & -S \\ R+S & -S & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}_{out} \\ \dot{\omega}_{MG2} \\ F \end{bmatrix} = \begin{bmatrix} -T_{load} \\ T_{MG2} \\ 0 \end{bmatrix} \end{cases} \tag{8}$$

Mode4 (Power Split):

$$\begin{bmatrix} I_{MG1}+I_e & 0 & 0 & -R \\ 0 & \frac{mr^2}{K^2} & 0 & R+S \\ 0 & 0 & I_{MG2} & -S \\ -R & R+S & -S & 0 \end{bmatrix} \begin{bmatrix} \dot{\omega}_{MG1} \\ \dot{\omega}_{out} \\ \dot{\omega}_{MG2} \\ F \end{bmatrix} = \begin{bmatrix} T_{MG1}+T_e \\ -T_{load} \\ T_{MG2} \\ 0 \end{bmatrix} \tag{9}$$

To construct models for the remaining five input-split and five output-split configurations, we can apply existing methods developed for the 2-PG design [4] by altering only the left hand side matrices. More specifically, only the last column and last row need to be changed based on gear connection [3].

In addition to the powertrain dynamics, a simple battery state-of-charge (SOC) model is developed as follows using the equivalent circuit model.

$$S\dot{O}C = -\frac{V_{oc} - \sqrt{V_{oc}^2 - 4P_{batt}R_{batt}}}{2R_{batt}Q} \tag{10}$$

where V_{oc} is the open circuit voltage, R_{batt} is the internal resistance of the battery, Q is the battery capacity, and P_{batt} is net power drawn by the two electric machines, which is described by

$$P_{batt} = T_{MG1}\omega_{MG1}(\eta_{MG1})^k + T_{MG2}\omega_{MG2}(\eta_{MG2})^k \tag{11}$$

where η_{MG1} and η_{MG2} are efficiencies of the two electric machines, and k takes the value of 1 when generating and -1 when motoring.

V. Optimal Energy Management

The optimal energy management problem is formulated to optimize the fuel consumption in given driving cycles. DP is used to solve this optimization problem to study the effect of adding clutches to enable more operating modes on single-PG configurations. The states and control variables used in DP is summarized in Table III, and the cost function is defined in Eq. (12). Details are provided in below.

TABLE III
STATES AND CONTROL VARIABLES IN DYNAMIC PROGRAMMING

Vehicle Configuration	States	Control variables
Input-split	ω_e, SOC	T_e, T_{MG1}, $Mode$
Output-split	ω_{MG1}, SOC	T_e, T_{MG1}, $Mode$

As shown in Table III, the states of the input-split and of the output-split configurations are not identical because of the different ways powertrain elements are connected to the PG. Nevertheless, for both input- and output-split configurations, the speed of the output shaft (ω_{out}) is specified by the drive cycle, and then the speeds of other powertrain elements are calculated using Eq. (1). Both input- and output-split configurations have the same control variables, the engine torque, MG1 torque and the Mode. The torque of MG2 is then calculated based on the power balance. When one or more clutch is closed, the PG will lose one degree of freedom, which will result in reduced number of state variables and/or control variables.

$$\min\left\{ J = \sum_{k=0}^{N-1} (FC_k + \beta \cdot \Delta Mode_k + \gamma \cdot \Delta SOC_k) \right\} \tag{12}$$

The cost function in Eq. (12) has three terms: fuel consumption (FC), mode shift penalty and SOC penalty. The mode shift penalty is necessary to ensure that the number of mode shifts is not excessive, and the SOC penalty ensures that the vehicle uses the battery efficiently when in pure electric modes (Mode 1 and 2). The penalty weights, β and γ, are chosen to be small so that the fuel consumption is still the dominating term in the optimization. In addition, all the fuel economy results shown in this paper have been corrected for SOC. Therefore, the small penalty added by the γ term should have little impact to the simulation results.

VI. Case Studies

In this section, we demonstrate the proposed analysis method on two cases: one is the input-split configuration used by the Toyota Prius (Fig. 3-(d)), and the other is the output-split configuration used by the Chevy Volt (Fig. 5). The FUDS and HWFET drive cycles are used to test their fuel economy. We allow blended operation instead of forcing charge depletion operations so the dynamic programming can select the most efficient operation freely among the four operating modes. The parameters of powertrain elements for both vehicles are acquired from [29] and summarized in Table IV. Efficiency maps for the engine and electric machines are acquired from [30]. However, the efficiency maps for the powertrain elements may not be identical to those on the production Prius and Volt, and thus our analysis should not be interpreted as a comparison for the two vehicles, but rather a demonstration of how we attempt to improve the design of each HEV powertrain configuration.

TABLE IV
PARAMETERS OF POWERTRAIN ELEMENTS
(IS: INPUT-SPLIT; OS: OUTPUT-SPLIT)

Parameters	MG1	MG2	Engine
Max. Speed (rpm)	12000 (IS) 6000 (OS)	12000 (IS) 9500 (OS)	4000
Max. Torque (Nm)	200(IS) 200 (OS)	200 (IS) 370 (OS)	102@4000 rpm
Max. Power(kW)	42 (IS) 55(OS)	60 (IS) 110 (OS)	43
Battery size(kWh)	8		
Planetary Gear Ratio (R:S)	2.6:1 (IS); 2.24:1 (OS)		
Final Drive Ratio	3.95 (IS); 2.16 (OS)		
Vehicle Mass(kg)	1300(IS);1700(OS)		

A. The Toyota Prius configuration

The powertrain configurations of the original Prius and the conceptual design, the Prius^{++}, are shown in Fig. 8. The Prius^{++} is obtained by adding three clutches to the Prius powertrain while keeping everything else intact. The original Prius has no clutch and thus only operates in Mode 4, while the Prius^{++} can operate in any of the four modes (and the DP solution will choose the best control inputs, including the mode). The optimal fuel consumptions of these two configurations are listed in Table V. It can be seen that the Prius^{++} achieves a fuel economy improvement of 18% over that of Prius in the urban (FUDS) cycle, which shows the benefit of the additional operating modes.

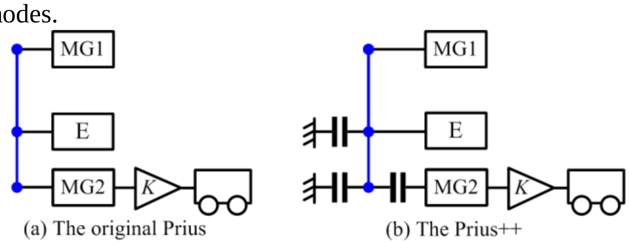


Fig. 8. The configuration sketches of the original Prius and 'Prius++'.

TABLE V
OPTIMAL FUEL CONSUMPTION OF PRIUS/PRIUS+/PRIUS++ IN FUDS CYCLE

Vehicle	Fuel Consumption (g)
Prius (only Mode 4)	115.3
Prius^{+} (Mode 2 & 4)	96.7
Prius^{++} (all four modes)	96.2

Further analysis on the DP solutions shows that the Prius^{++} mainly operates in Modes 2 and 4 (see Fig. 9). The other two modes are rarely used. Therefore, we looked into an alternative design, the Prius^{+}, which has only one clutch to switch between Mode 2 and Mode 4. The Prius^{+} configuration is shown in Fig. 10. The fuel economy of Prius^{+} is also listed in Table V, and its DP solution is shown in Fig. 11. Comparing Fig. 9 and Fig. 11, one can find that the optimal mode selection of the Prius^{+} is very similar to that of the Prius^{++}, and the fuel economy is very close. This confirms our hypothesis that the simplified one-clutch design can achieve near-optimal fuel economy.

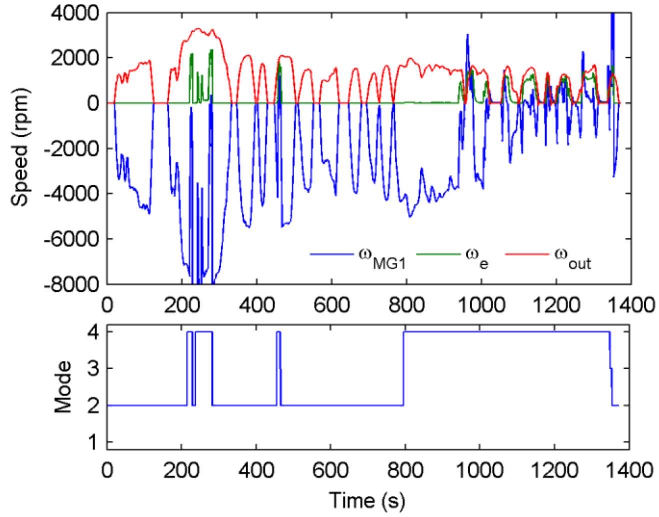


Fig. 9. The speeds of powertrain elements and optimal mode selection of the Prius++ in the FUDS cycle.

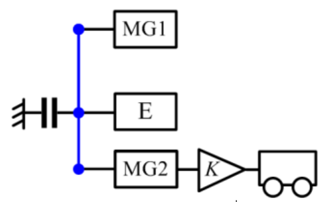


Fig. 10. The configuration sketch of the 'Prius^{+}'.

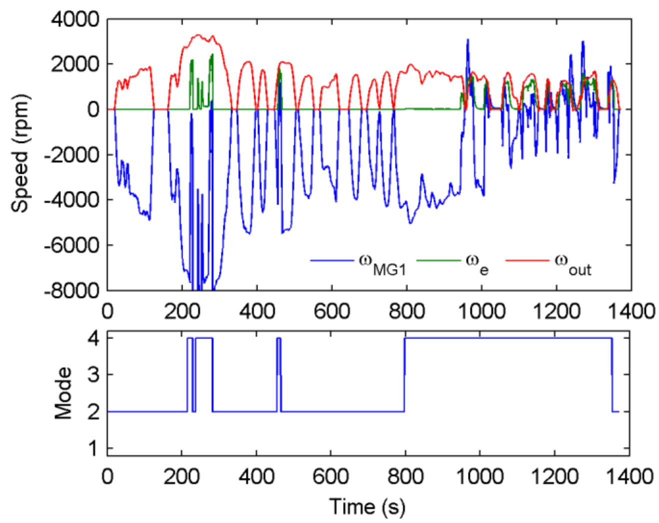


Fig. 11. The speeds of powertrain elements and optimal mode selection of the Prius+ in the FUDS cycle.

We also examine the fuel economy of the three vehicles (Prius, Prius^{+} and Prius^{++}) in highway driving using the HWFET cycle. The optimal fuel consumptions for all three configurations are very close (see Table VI), which means the benefit of additional modes in highway driving is minimal.

TABLE VI
OPTIMAL FUEL CONSUMPTION OF PRIUS/PRIUS+/PRIUS++ IN HWFET CYCLE

Vehicle	Fuel Consumption (g)
Prius (only Mode 4)	314.8
Prius^{+} (Mode 2 & 4)	312.7
Prius^{++} (all four modes)	310.0

In summary, adding a clutch (see Fig. 10) to enable the extra Mode 2 on the Prius configuration is beneficial for urban driving when the vehicle has a rather large battery (i.e. the vehicle is more a plug-in HEV rather than a stand-along HEV), but has little benefit on fuel economy for highway driving.

B. *The Chevy Volt configuration*

The original design of the Chevy Volt has three clutches and can operate in any of the four modes shown in Fig. 5. The DP solution of the Volt in the FUDS drive cycle is shown in Fig. 12. We observed that Modes 1, 3 and 4 are frequently used. A closer observation further shows that Mode 3 can replaced by Mode 1 because much of the Mode 3 operation has the engine speed at zero, which is essentially equivalent to the pure EV drive. The analysis leads to the alternative configuration which only switches between these two modes, named the Volt $^-$ (shown in Fig. 14-(b)). Our analysis confirms that Mode 1 is more efficient than Mode 2 in urban driving, and Mode4 is more efficient than Mode3 in almost all conditions. The fuel consumptions of these two powertrains in the FUDS cycle are shown in Table VII and they are very close.

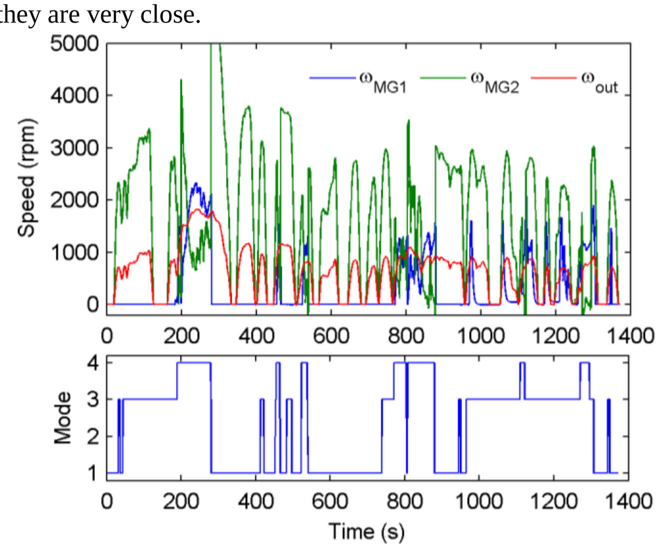


Fig. 12. The speeds of powertrain elements and optimal mode selection of the Volt in the FUDS cycle

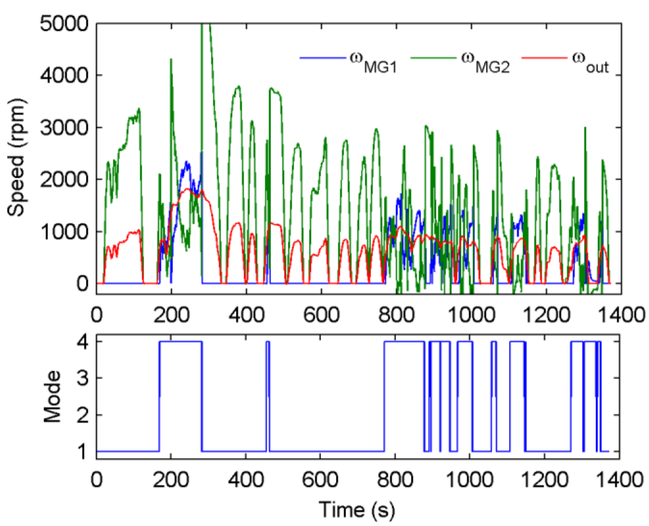
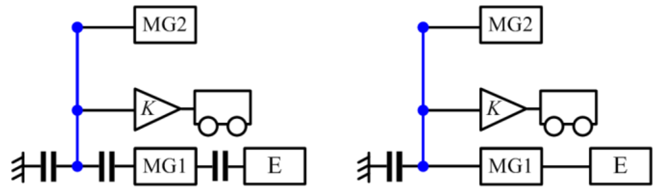


Fig. 13 The speeds of powertrain elements and optimal mode selection of the Volt- in the FUDS cycle



(a) The original Volt (b) The Volt –

Fig. 14. The configuration sketches of the original Volt and "the Volt-".

TABLE VII
OPTIMAL FUEL CONSUMPTION OF VOLT/VOLT- IN FUDS CYCLE

Vehicle	Fuel Consumption (g)
Volt (all four modes)	167.0
Volt- (Mode 1 & 4)	168.5

The HWFET cycle is again used to examine the fuel economy in highway driving. The optimal fuel consumptions of the original Volt and Volt$^-$ are shown in Table VIII. Again, the Volt$^-$ has fuel economy very close to that of the original Volt.

TABLE VIII
OPTIMAL FUEL CONSUMPTION OF VOLT/VOLT- IN HWFET CYCLE

Vehicle	Fuel Consumption (g)
Volt (all four modes)	379.2
Volt$^-$ (Modes 1 & 4)	379.6

In summary, the simplified design in Fig. 14-(b) with one clutch and two operating modes (Modes 1 and 4) achieves fuel economy similar to that of the original Volt for both urban and highway driving, while eliminating two clutches.

VII. CONCLUSION

We present a thorough analysis of all possible configurations of power-split hybrid powertrain using a single PG. the analysis shows there can be 12 different configurations, in which 6 are input-split the other 6 are output-split. Each input-split configuration can have as many as 6 clutches added to achieve 8 operating modes, and each output-split configuration can have as many as 5 clutches added to achieve 8 modes. However, in both input- and output-split configurations, only 3 clutches are actually needed to achieve the 4 useful modes.

We further focused on the two configurations used by the Toyota Prius and Chevy Volt; the first being an input-split configuration and latter an output-split configuration. Optimal fuel consumptions were found using DP with powertrain parameters/efficiency maps that are similar (although not identical) to the two vehicles. The DP solutions show that, in general, having more operating modes improves the fuel economy because those additional modes provide more flexibility for powertrain elements to work efficiently. However, not all modes are necessary; some modes are rarely used, so they may be eliminated without significantly degrading the fuel economy. This has been observed in both our Prius$^+$ and Volt$^-$ designs, which are configurations derived from the original Prius and original Volt.

The Prius$^+$, with one clutch and two modes (Mode 2 and 4), achieves similar fuel economy in the FUDS cycle as the Prius^{++}, with three clutch and all four modes. In addition, the Prius$^+$ is significantly better than the original Prius, with no clutch and only one mode (Mode 4). However, adding modes has little fuel economy benefit for the HWFET cycle. The Volt', with one clutch and two modes (Mode 1and 4), achieves almost the same fuel economy in both FUDS and HWFET cycle as the original Volt, which has three clutches and four modes.

Our analysis indicates that the decisions of adding or eliminating clutches for designing single-PG HEV configurations are not intuitive and require a series of systematic analysis and control optimization. We also confirm that this systematic analysis on single-PG HEV configurations is valuable because the performance indeed can be improved. However, the analysis proposed in this paper ignores several factors; for example, the gear efficiency, acceleration performance, and series charging while parking are not considered, and this paper focused on performances of fuel economy in FUDS and HWFET cycles only. Some of the parameters are based on assumed values and may not be close to the real parameters in those two vehicles. Nevertheless, the systematic mode analysis and DP optimization for identifying the best configuration still provide useful insights in the design and analysis of power-split hybrid vehicles.

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